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## ELECTRONIC STRUCTURE AND PROPERTIES

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# Detecting Helical Andreev Edge States by Shot-Noise Measurements

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It has been theoretically argued that a combination of two electronic techniques, measurements of conductivity and shot noise spectra, can be an effective method for detecting helical Andreev edge states in quantum coherent conductors. To implement this methodology in practice, we propose an interferometric phase-sensitive configuration consisting of two independent scanning probe tips, normal and superconducting, and show that the corresponding edge current is strongly dependent on the applied magnetic field.

**Key words:** helical Andreev edge currents, two-probe scanning setup, superconducting tip, electronic shot-noise spectra.

Теоретично показано, що комбінація двох електронних методів, вимірювання спектрів провідності і електронного дробового шуму може бути ефективною для виявлення гвинтових андріївських крайових станів у квантових когерентних провідниках. Щоб реалізувати цю методологію практично, ми пропонуємо інтерферометричну фазочутливу конфігурацію, що складається з двох незалежних сканувальних зондів, нормального і надпровідного, і показуємо, що відповідний крайовий струм сильно

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залежить від прикладеного магнетного поля.

**Ключові слова:** гелікоїдальні андріївські крайові струми, двозондовий сканувальний пристрій, надпровідне вістря, спектри електронного дробового шуму.

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## 1. INTRODUCTION

Discovery of the integer quantum Hall effect in strong magnetic fields and then their topological electronic phenomena led to a new revolution in understanding the ubiquitous topological structure of quantum matter. As known, the topological properties are characterized by a global integer quantity invariant under smooth geometrical deformations or material changes. Therefore, the related physical characteristics have some form of topological protection or, by other words; they are guaranteed to withstand any continuous disturbance. In particular, it relates edge states with a strong immunity to distortions of the sample unlike non-topological boundary states that are fragile. Although the topological edge modes originate from the internal topology, they exhibit properties, forbidden in the bulk.

Canonical example of such phenomenon is the mentioned above quantum Hall effect where electrons are moving along cyclotron orbits in the bulk while the quantum transport is dominated by unidirectional propagating states localized at the boundaries and is robust against local perturbations, too [1].

Quantum Hall edge states in proximity with an $s$-wave superconductor can give rise to electron-hole hybrid states called Andreev edge states. Because of the absence of a backscattering channel, an electron moving along such state can be either Andreev reflected as a hole, or transmitted as an electron to the superconductor. In this paper, we compute the differential conductance and the noise spectra associated to this partitioning in the two outgoing channels. In our interferometric setup for probing long-range conducting edge modes by the powerful methods of superconducting proximity effect, there are two independent probe tips, one normal and one s-wave superconducting. Along these lines, in the paper [2], we showed that such a device is able to realize quantum interference effect of quasiparticle excitations moving in opposite directions along metallic-like one-dimensional edge channels and therefore, it can be useful for revealing the origin of anomalous conducting edge states in quantum materials. Furthermore, the helical states might also be advantageous to analyze the entanglement of electrons injected from a singlet $s$-wave superconductor.

Recently, a new intriguing physics is found at the boundary between two active fields of research, non-Hermitian systems and topological effects [3]. In quantum samples, broken Hermiticity means a lack of probability conservation or a loss of quantum information. At the same time, all practically important systems are non-Hermitian due to the inevitable coupling to degrees of freedom outside the system of interest. For classical-wave (electromagnetic or mechanical) phenomena, non-Hermiticity is related to a lack of conservation of the wave energy. In this case, novel effects are observed, using active mechanisms that couple local degrees of freedom of artificial materials to the environment. Such topological classical-wave systems allow for robust point-to-point energy guiding with an engineered immunity to disorder or geometrical imperfections. That is why now there is a surge of activity in the study of non-conservative topological systems [3, 4].

Moreover, it is shown that, under some general conditions, topological insulators, non-Hermitian in the bulk, *i.e.*, with complex bulk energy spectra, can have chiral boundary states with an effective Hermitian Hamiltonian [5]. In particular, it relates Hermitian helical hinge states in three-dimensional second-order topological insulators. In this case, the transmission coefficient of each chiral channel in non-Hermitian topological insulators is precisely unity, in contrast to the non-quantized value of a general non-Hermitian topological insulator and such Hermitian nature of the boundary states is insensitive to smooth changes of material parameters and cannot be destroyed unless the system undergoes a topological phase transition [5].

A key challenge to discriminate between pure quantum and classical phenomena in the topologically protected charge transport concerns experimental techniques able to explore and/or to control quantum properties of the edge currents. In this paper, we propose to realize this task by performing shot-noise measurements in metallic-like edge channels with a phase-sensitive setup with normal ($N$) and superconducting ($S$) tips described in our previous paper [2].

In mesoscopic devices, noise arises both at equilibrium, leading to thermal noise, and out of equilibrium, when partitioning of electrons at a scatterer generates shot noise [6]. The recent progress in the development of scanning tunnelling shot noise spectroscopy using a scanning tunnelling microscope with a tip used to simultaneously measure the current-voltage characteristics and the zero-frequency shot noise has proved to be extremely useful to gain insights on the nature of charge carriers [7]. Electronic shot noise is known to be a useful informational source because it depends on the distribution of transmission channels, which determines quantum transport in the framework of the Landauer formalism. In particular, such measurements have provided first experimental proves of fractional charges [8].

They are very sensitive to particle-hole symmetry, an evident property of Andreev edge states carrying superconducting correlations across the non-decaying modes. It has been one of the main reasons for the $S$ tip in our device [2].

## 2. MODELLING

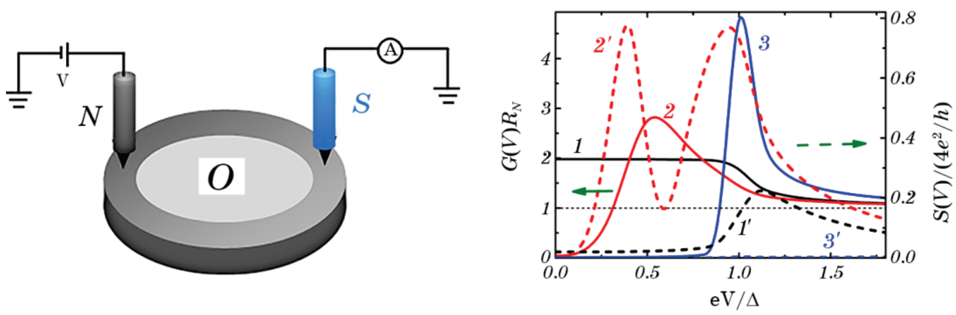
To elucidate the difference between the conductive boundary and the insulating bulk contributions to the transport characteristics as well as quantum versus classical character of the edge currents, we discuss a two-dimensional Corbino disk with the uniform density of mobile carriers in the quantum Hall effect plateau. To calculate the current-versus-voltage curves for such setup, we use our general scattering-like approach to transport characteristics of a superconductor-based device coupled to two electron reservoirs, left ($L$) and right ($R$), with different chemical potentials $\mu_L$ and $\mu_R$ as is explained in the work [2].

Following our methodology [9], we interpret the charge transmission across the setup shown in Fig. 1, *a* as a sequence of an infinite number of interface scattering events including Andreev electron-hole and vice versa transformations at the boundary with the $S$ tip. As is explained above, for an electron (e) entering the $N$ tip from the $L$ reservoir there are two possibilities, to be scattered back as an electron with the probability amplitude $R^{\text{ee}}(\varepsilon)$ or as a hole (h) with the probability amplitude $R^{\text{eh}}(\varepsilon)$:

$$\begin{aligned} R^{\text{eh}} &= t_O^{\text{e}} r^{\text{eh}} t_O^{\text{h}} (1 + r_O^{\text{h}\leftarrow} r^{\text{he}} r_O^{\text{e}\leftarrow} r^{\text{eh}} + \ldots) = t_O^{\text{e}} r^{\text{eh}} t_O^{\text{h}} / (1 - r_O^{\text{h}\leftarrow} r^{\text{he}} r_O^{\text{e}\leftarrow} r^{\text{eh}}); \\ R^{\text{ee}} &= r_O^{\text{e}\rightarrow} + t_O^{\text{e}} r^{\text{eh}} r_O^{\text{h}\leftarrow} r^{\text{he}} t_O^{\text{e}} (1 + r_O^{\text{h}\leftarrow} r^{\text{he}} r_O^{\text{e}\leftarrow} r^{\text{eh}} + \ldots) = \\ &= r_O^{\text{e}\rightarrow} + t_O^{\text{e}} r^{\text{eh}} r_O^{\text{h}\leftarrow} r^{\text{he}} t_O^{\text{e}} / (1 - r_O^{\text{h}\leftarrow} r^{\text{he}} r_O^{\text{e}\leftarrow} r^{\text{eh}}). \end{aligned} \tag{1}$$

Here $\varepsilon$ is energy of quasiparticles. In Eq. (1) $t_O^{\text{e(h)}}$ are probability amplitudes for an electron (e) or a hole (h) transmitted across the normal-state edge mode, $r_O^{\text{e(h)}\rightarrow}$ and $r_O^{\text{e(h)}\leftarrow}$ are probability amplitudes for quasiparticles arriving from the tip $N$ ($\rightarrow$) and the tip $S$ ($\leftarrow$) and scattered back from the $O$ region shown in Fig. 1. The presence of an $S$ tip introduces additional Andreev-retroreflection $r^{\text{eh}}$ and $r^{\text{he}}$ amplitudes when an electron (hole) incident from the $O$ side to the interface with a superconductor is scattered back into a hole (electron) state, respectively. The latter events by which normal current is transferred to a supercurrent are characterized by amplitudes $h(\varepsilon) = \text{sign}(\varepsilon)\sqrt{\varepsilon^2 - \Delta^2}$ for $|\varepsilon| > \Delta$ and $h(\varepsilon) = i\sqrt{\Delta^2 - \varepsilon^2}$ for $|\varepsilon| < \Delta$, where $\Delta$ is the energy gap of the $s$-wave $S$ tip [9].

The probability amplitudes $t_O^{\text{e(h)}}$, $r_O^{\text{e(h)}\rightarrow}$, and $r_O^{\text{e(h)}\leftarrow}$ are found in the work [10] using modified Griffith boundary conditions at the junction nodes. In the external magnetic field **H** perpendicular to Corbino disk plane, electrons and holes propagating along a loop from the left to the right node pick up an additional phase $\pm 2\pi\Phi L_{1,2} / (\Phi_0 L)$ depending on




*a* *b*

**Fig. 1.** The spectroscopic device (*a*) and expected conductance $G(V)R_N$ (solid lines) and the normalized shot noise $S(V)/(4e^2/h)$ (dashed lines) spectra in the presence of an external magnetic field **H** (*b*). Magnetic flux through the quantum loop is 0, $0.01\Phi_{0s}$, and $0.1\Phi_{0s}$ (curves *1*, *2*, and *3*), $\Phi_{0s} = h/2e$ is the elementary quantum flux in a superconducting state, $R_N = (dI(V)/dV)_N$ is the resistance of the sample in the normal state, $\Delta$ is the energy gap of the $S$ tip. $L_1 = L_2$, $\eta = 1$.

the carrier charge and a clockwise or anticlockwise motion in the two segments of the loop with lengths $L_1$ and $L_2$, $\Phi_0 = h/e$ being the elementary flux quantum for a normal circuit [10]. The sum $L = L_1 + L_2$ is the loop circumference. The difference in the amplitudes $t_O^{e(h)}$, $r_O^{e(h)\rightarrow}$, and $r_O^{e(h)\leftarrow}$ for holes and electrons originates from that between wave numbers $k^{e(h)}(\varepsilon) = k_F \pm \varepsilon / (\hbar v_F)$, $k_F$ and $v_F$ are Fermi wave number and velocity, respectively. An impact of the inequality $k^e(\varepsilon) \neq k^h(\varepsilon)$ depends on the value of the dimensionless parameter $\eta = L\Delta / (\hbar v_F)$.

If all scattering parameters are known, we can calculate the probability amplitudes $R^{ee}(\varepsilon)$ and $R^{eh}(\varepsilon)$ as functions of the energy $\varepsilon$ and, at last, the current $I$, differential conductance $dI/dV$, and zero-frequency shot noise $S$ as functions of the applied voltage bias $V$ at zero temperature. In the latter case, we limit ourselves to the low-frequency limit of the spectral density $S_I(V)$, which is the Fourier transform of the correlation function $S(t-t') = \langle \Delta I(t) \Delta I(t') \rangle$ with $\Delta I(t) = I(t) - \langle I(t) \rangle$, the current fluctuation depending on time $t$:

$$S(V) = \frac{4e^2}{h}\int_0^{eV}\left[\left|R^{ee}(\varepsilon = eV)\right|^2\left(1-\left|R^{ee}(\varepsilon = eV)\right|^2\right) + \left|R^{eh}(1 = eV)\right|^2 \times \right.$$
$$\left. \times\left(1-\left|R^{eh}(1 = eV)\right|^2\right)\right]d\varepsilon + \frac{8e^2}{h}\int_0^{eV}\left[\left|R^{ee}(\varepsilon = eV)\right|^2\left|R^{eh}(\varepsilon = eV)\right|^2\right]d\varepsilon. \quad (2)$$

This result will be compared with the normalized differential conductance spectra $G(V) = (dI(V)/dV)_S/(dI(V)/dV)_N$ that is calculated in the paper [2]:

$$G(V) = 1 - \left|R^{ee}(\varepsilon = eV)\right|^2 + \left|R^{eh}(\varepsilon = eV)\right|^2. \quad (3)$$

## 3. NUMERICAL RESULTS AND DISCUSSION

To show the advantage of shot-noise measurements performed together with those of the differential conductance, we have performed numerical calculations for the same parameters as in our paper [2] where we discussed only effects in the $G(V)$ dependence. The voltage $V = (\mu_L - \mu_R)/e$ applied to the setup generate the charge flow across it. The presence of the $S$ tip strongly enhances sensitivity of the shot-noise spectra $S_I(V)$ to the charge transmission regime across the quantum ring. Observation of this effect in external magnetic fields together with related conductance spectra that is shown in Fig. 1 will prove the presence of quantum correlations in the charge flow localized at the edges of the sample. Even more, in classical-wave systems, the shot noise is absent, and its measurement will unambiguously clarify the nature of the currents under discussion.

As is stressed in the paper [11], electrical conductance measurements have a limited scope in identifying hybrid Andreev edge states, which form the basis for realizing various topological excitations in normal-state edge—superconductor contacts. To detect the exotic states, the authors [11] measured shot noise together with the conductance spectrum in a graphene-based quantum Hall—superconductor junction. They declared a closer agreement of experimental results with the scenario of uniform phase averaging that demonstrated crucial involvements of static disorder and inelastic processes along the current-carrying boundary modes even at very low temperature. We have proposed another type of the shot-noise spectroscopic devices based on two independent N and S tips, charge transport between which is carried out along topologically protected metallic-like edge channels.

Generally, a combination of two electronic methodologies in quantum coherent conductors, conductance and shot noise spectra, has been extensively used for extracting an information on quantum transport. In particular, it has been done in studies of the fractional quantum Hall effect [12], spin-polarized quantum transport [13–16], Kondo effect [17], electron-vibration interaction [18, 19], *etc*. We expect that similar two-probe experiments together with conductance measurements can be applied as a detection tool in the search for topologically protected excitations in novel quantum materials with a non-Hermitian interior. The interest in such phases stems from their robust transport properties, which offer fascinating possibilities for the passage of quantum information exploiting topological immunity to defects and disorder [20].

Concerning an experimental implementation of the proposed experiments, we can refer to the paper [21] where two-probe scanning tunnelling experiments are performed at cryogenic temperature of 4.5 K with electrochemically etched tungsten tips independently driven on a surface and probing distance below 50 nm. In our opinion, there will be no problems to replace normal-metal tips in such device with superconducting ones and to use a related well-developed technique [22]. In this case, such device will be suitable for the experiments proposed above.

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