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## Application of Dimensional Analysis for Stable Dry Drawing Process Designing

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Using the analysis of dimensions, the generalized method of designing rational modes of multiple drawing of wire with dry soapy technological lubricant is improved theoretically and experimentally, taking into account the friction mode in the drawers, other process parameters and determining the acceptable multiplicity of metal flow deformation in the mill. An algorithm is developed for calculating the rational number of rods passes through the dies during flow drawing, the use of which allows, in its implementation in practice, to ensure the stability of the technological process with the absence of wire breaks with the minimum necessary (optimal) number of metal deformation cycles and the corresponding energy consumption. The obtained results are consistent with the practice of industrial drawing with the use of dry technological lubricants on a soap base and can be used in the development of new ones and in checking the rationality of existing technological drawing processes. In the future, it is advisable to develop a program for automatic calculation according to the above algorithm for the rational number of passes during dry drawing, as well as use the presented approach to expand the initial statistical database, taking into account the properties of other steel grades that are deformed in the processes of dry and wet drawing.

**Key words:** metrology, dimensional analysis, measurements, wire drawing, process stability.

З використанням аналізу розмірностей теоретико-експериментальним шляхом удосконалено узагальнену методу проектування раціональних режимів багатократного волочіння дроту із сухим мильним технологіч-

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ним мастилом при урахуванні режиму тертя у волоках, інших параметрів процесу та визначенням доцільної кратності поточного деформування металу на стані. Розроблено алгоритм розрахунку раціональної кількості пропусків заготовки крізь волюки при поточному волочінні, використання якого дозволяє при своїй реалізації на практиці забезпечити сталість технологічного процесу з відсутністю обривів дроту за мінімально необхідної (оптимальної) кількості циклів деформації металу та відповідних енерговитрат. Одержані результати узгоджуються з практикою промислового волочіння з використанням сухих технологічних мастил на мильній основі та можуть бути використані при розробці нових та перевірці раціональності існуючих технологічних процесів волочіння. У перспективі доцільною є розробка програми автоматичного розрахунку за наведеним алгоритмом раціональної кількості пропусків при сухому волочінні, а також використання представленого підходу для розширення вихідної статистичної бази даних з урахуванням властивостей інших марок криць, які деформуються у процесах сухого і мокрого волочіння.

**Ключові слова:** метрологія, аналіза розмірностей, вимірювання, волочіння дроту, стабільність технологічного процесу.

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## 1. INTRODUCTION

Hot-rolled wire rod, which is pre-produced on wire- and light-section mills is used as a blank for single and current multiple steel wire drawing. Such a wire rod carries scale layer, which is usually removed by pickling in an acid solution before drawing. After neutralization, a dried sub-lubricant coat is formed on the wire rod, which usually consists of thin ‘soft’ layers of iron oxide and lime, borax, *etc.* This provides better adhesion of process when lubricant is applied to the wire rod surface before deformation in wire dies.

Sometimes the wire rod instead of pickling is subjected to machining (alternating bending) in a roller scale breaker [1]. This operation increases contamination of the lubricant with scale dust, impairs its adhesive and antifriction properties [1]. To prevent this, such a rod is rubbed with wipers in the line, and sometimes a dry sub-lubricating layer of powdered appropriate materials is applied on its surface [2].

The wire rod is usually subjected to drawing in the flow on the drawing mills of multiple deformations in several passes. At the same time, they try to minimize the number of passes for saving resources, but it increases the required traction force (tension) of drawing process. The level of this force can approach the strength of the wire, which accompanied by its break [1]. Therefore, the main limitation for reducing the number of passes is the condition of no wire breaks during drawing.

When drawing wire with a diameter of 0.8–10.0 mm (and sometimes larger) the process lubricant is usually used a dried powdered soap (fat-

ty acid salt) of natural or synthetic origin without functional additives or with them (lime, talc, mica, *etc.* [1]).

In the drawing mill, such lubricant is stored in containers installed in front of the dies. Its involvement to the deformation centre of the following die carries out by a moving rod passing through the bulk mass of the powder, due to the adhesion of the latter to the metal surface [1].

The amount (thickness) of lubricant together with the height of the microrelief on the wire determine the mode of friction in the deformation zone, wear of the dies, drawing force conditions, product surface quality, and ultimately efficiency and cost-effectiveness of the production process [3]. Therefore, optimization of the number of passes under specific drawing conditions connected, in particular, with determination of the friction mode, and becomes an important prerequisite for designing and implementation of appropriate rational technology.

The friction mode is evaluated by the ratio:

$$\theta_i = \frac{\xi_{av.i}}{R_{a.av.i}}, \quad (1)$$

where  $\xi_{av.i}$  — is the average thickness in the centre of deformation of the lubricating layer, which according to the condition of longitudinal continuity of the latter is associated with the thickness  $\xi_{1.i}$  by simple geometric ratios at the exit of the  $i$ -th die by relation [3]:

$$\xi_{av.i} \approx \frac{1}{2}(\sqrt{\mu_i} + 1)\xi_{1.i}, \quad (2)$$

the metal elongation factor (coefficient of elongation) while drawing in the  $i$ -th die:

$$\mu_i = (d_{0.i} / d_{1.i})^2, \quad (3)$$

$i$  is the number of die in the direction of  $n$ -fold drawing;  $1 \leq i \leq n$ ;  $d_{0i}$ ,  $d_{1i}$  are diameters of the wire at the entry (hereinafter—index «0») and at the exit (hereinafter—index «1») of  $i$ -th die, respectively; average height of roughness is:  $R_{a.av.i} = 0.5(R_{a.0.i} + R_{a.1.i})$ ;  $R_{a.0}$ ,  $R_{a.1}$  are heights of microrelief at entry of metal to the  $i$ -th die and exit from it respectively, while value  $\xi_{1.i}$ , in its turn is determined by the formula:

$$\xi_{1i} = m_{1i} / \rho_{1i}, \quad (4)$$

where  $m_{1i}$  (kg/m<sup>2</sup>) is the relative weight of lubricant rests on the wire after leaving the die,  $\rho_{1i}$  is related density of the lubricant.

Simple transformation on formulas (1)–(3) considering provides:

$$\theta = \frac{m_{1i}(\sqrt{\mu} + 1)}{\rho_{1i}(R_{a,0,i} + R_{a,1,i})}. \quad (5)$$

Value  $\theta_i$  in the range of 3–4 indicates on friction mode close to the liquid mode in deformation zone, thus determining the minimum values of  $f_i$  in the range that is generally inherent to the corresponding conditions of drawing process. This mode of drawing is desirable to minimize energy consumption, despite natural increase in lubricant consumption. Level  $\theta_i < 1$  indicates approach of friction mode to the boundary one with increase of  $f_i$ . Level  $\theta_i = 1$ –3 indicates presence of mixed friction mode, and  $\theta_i \rightarrow 0$  can bring the friction mode closer to dry one with the appropriate values  $f_i \rightarrow 0.5$  [4].

## 2. THEORETICAL METHODOLOGY

Known studies of these aspects relate both to the calculation of force conditions under  $n$ -fold drawing [1, 5], and determination on technological factors affecting of soap lubricant amount residues on the wire after drawing and formation of friction mode in dies [3].

The relevant research under the guidance of one of the authors of this article was earlier carried out based on statistical processing of data obtained (with participation of Dr. of Eng. Sci. I. B. Buravlev, candidate of engineering sciences Yu. B. Sigalov, and engineer L. O. Didenko) in Dnipropetrovsk Metalware Production Association (now—‘DNIPROMETYZ’) during of mass wire production made of low-carbon wire-rod made of steels: Ст0М (initial yield strength  $\sigma_{T,init} = 340$ –380 MPa), Ст3кп ( $\sigma_{T,init} = 268$ –380 MPa), Ст10кп ( $\sigma_{T,init} = 259$ –267 MPa), Ст2кп ( $\sigma_{T,init} = 230$ –240 MPa), Ст20кп ( $\sigma_{T,init} = 290$ –300 MPa), CB08Г2С ( $\sigma_{T,init} = 385$ –390 MPa),—having initial diameter  $d_{init} = 6.40$ –6.70 mm and microrelief height  $R_{a,init} = 2.0$ –6.4  $\mu\text{m}$ , produced by hot rolling process with application of accelerated cooling method in Stelmor line at PJSC ‘Arselor MITTAL Kryvyi Rig’ [6], also steels Ст3кп after natural air cooling ( $\sigma_{T,init} = 185$ –190 MPa) and Ст3кп after forced active (‘intensive’) cooling from rolling temperature down to 550°C ( $\sigma_{T,init} = 480$ –540 MPa).

Wire samples with diameters  $d_i = 5.90$ –2.45 mm after drawing at a speed  $V_i = 0.6$ –15 m/s in  $n = 1$ –6 passes respectively, and with diameters  $d_i = 2.95$ –1.25 mm after deformation in  $n = 1$ –7 passes of semi-finished annealed rod of diameter  $d_{init} = 2.95$ –3.95 mm were taken for research. A total of 100000 products were made and 50–100 samples of wire were taken for each steel.

The mechanical properties of the wire samples were determined during standard rupture tests, the height of the microrelief—using profilometer of model ‘283’, and the amount of lubricant residue on the

same—by weight method [3] using scale AДB-200 type.

The following lubricants were used:

- sodium soap powder (as a base) in the initial state without any additives (hereinafter—‘SP’ marked),
- sodium soap powder with addition of 10% talc (‘SPT’ marked);
- sodium soap powder with addition at its initial state of 15% lime after heat treatment in a special installation at a temperature up to 150°C (‘SPL’ marked).

When passing of processing wire through the deformation line in the drawing mill, its microrelief is smoothed, and the surface layer of lubricant is repeatedly thinned, subjected to heating (up to 200–500°C), compaction and may be incompletely transferred to the next die. As a result, in the containers before the dies, the lubricant is saturated with the products of wear and thermal decomposition, loses anti-friction efficiency, and is periodically (often in uncontrolled manner) replenished by a worker, usually in the first container with fresh lubricant. Therefore, samples of these lubricants were taken periodically for analysis with averaging of their composition during the working shift of the drawing mills. It is worth mentioning that in the course of research there was a significant scatter of all experimental data (values of coefficients of variation reached 0.2–0.3), which is due to the instability of the rod’s parameters and production conditions of the drawing processes.

The density of used lubricants in the first container in the drawing direction on average was: for SP  $\rho_{0SP} = 2690$  kg/m, for SPT  $\rho_{0SPT} = 2730$  kg/m, for SPL  $\rho_{0SPL} = 2900$  kg/m [3].

As a result, the density  $\rho_{1i}$  of lubricants residue on the wire towards the exit of the next  $i$ -th die increases with growth of cumulative metal elongation factor (cumulative coefficient of elongation):

$$\mu_{\Sigma i} = \left( \frac{d_{\text{init}}}{d_{1i}} \right)^2 \quad (6)$$

in accordance with the empirical relation:

$$\rho_{1i} = \rho_0 (1 + a(\ln \mu_{\Sigma i})^b), \quad (7)$$

where for lubricant SP,  $a = 0.340$ ,  $b = 0.583$ , for SPT  $a = 0.513$ ,  $b = 0.226$ , for SPL  $a = 0.345$ ,  $b = 0.198$  [3].

The formula of the view (6) is similar to equation (3) and is also used to determine the one-time coefficient of elongation in the  $i$ -th pass at  $d_{\text{init}} = d_{0i}$ .

The previously performed dimensional analysis and subsequent statistical data processing revealed the dependence of the relative lubricant amount  $m_i$  (kg/m<sup>2</sup>) on the wire surface on the drawing speed  $V_i$ ,

the initial (before  $i$ -th die) yield strength  $\sigma_{T0i}$  and the corresponding roughness of the metal  $R_{a0i}$ . In this case, the lubricant composition was factored into its density  $\rho_{1i}$ , the degree of deformation mediately—by introducing into the calculation formulas of the wire rod diameter  $d_{0i}$  at the entry to the next  $i$ -th die and wire diameter  $d_{1i}$  at its exit, and the change in height of the metal microrelief after and before of  $i$ -th die  $R_{a1i}/R_{a0i}$  that was depending on mass  $m_i$ , drawing stress  $\sigma_i$  in the  $i$ -th pass and the metal yield strength  $\sigma_{Ti}$  after deformation by empirical relation:

$$r = \frac{R_{a.1.i}}{R_{a.0.i}} = 0.57 \left( \frac{m_i}{\rho_{1i} R_{a.0.i}} \right)^{0.61} \left( \frac{\sigma_i}{\sigma_{Tav.i}} \right)^{0.65}, \quad (8)$$

where the ratio  $\sigma_i/\sigma_{Tav.i}$  is determined using the known theoretical formulas, for example [5]:

$$\frac{\sigma_i}{\sigma_{Tav.i}} = \ln \mu_i + 0.77\alpha + \frac{f_i \ln \mu_i}{\alpha + \alpha^2 + f_i \ln \mu_i} + \frac{0.8f_i\alpha\sigma_{Ti}}{\sigma_{Tav.i}(\alpha + f_i \ln \mu_i + 0.8\alpha)}, \quad (9)$$

$\alpha$  is half of the die cone angle (in common cases it is  $\alpha = 0.1 \pm 0.03$  radian for steel wire drawing process),  $f_i$  is friction factor,  $\mu_i = (d_{0i}/d_{1i})^2$  is metal elongation factor in the  $i$ -th pass,  $\sigma_{Tav.i}$  is average value of yield strength in the pass.

For the first pass the following shall be adopted [5]:

$$\sigma_{Tav.i} = \frac{1}{3}(\sigma_{T0i} + 2\sigma_{Ti}) \quad (10)$$

and for the other cases:

$$\sigma_{Tav.i} = \frac{1}{2}(\sigma_{T0i} + \sigma_{Ti}). \quad (11)$$

In absence of experimental values of  $\sigma_{Ti}$ , while calculating value of cumulative elongation factor  $\mu_{\Sigma i}$  before and after the  $i$ -th pass,  $\sigma_{Tav.i}$  can be determined using the empirical formula [5]:

$$\sigma_{Ti} = \sigma_{T.init} + u\sqrt{\ln \mu_{\Sigma i}}, \quad (12)$$

where dimensional factor  $u$  is equal: for steel Cт0M 412 MPa, for steel Cт3кп, cooled in Stelmor line 372 MPa, for steel Cт3кп after natural air cooling 380 MPa, for steel Cт3кп after forced active cooling from rolling heat to 550°C 258 MPa, for Cт10кп 415 MPa, for Cт2кп 413 MPa, for Cт20кп 510 MPa, for CB08Г2С 451 MPa.

Using the method of inverse calculation by formula (8) allowed to determine that  $f_i = 0.08$ – $0.24$ , and approximation of almost invariant

dependence  $f_i$  on criterion  $\theta_i$  under conditions of mixed and liquid friction in deformation zone for different lubricants creates relation [7]:

$$f_i = \frac{g}{h + q\theta_i}, \quad (13)$$

where  $g = 0.25$ ,  $h = 0.88$ ,  $q = 1.08$ .

If in equation (9) we use  $\sigma_{T1.i}$ , instead of value  $\sigma_{Tav.i}$  we will obtain formula [5]:

$$\frac{\sigma_i}{\sigma_{T1.i}} = \ln \mu_i + 0.77\alpha + \frac{f_i \ln \mu_i}{\alpha + \alpha^2 + f_i \ln \mu_i} + \frac{0.8f_i\alpha}{\alpha + f_i \ln \mu_i + 0.8\alpha}, \quad (14)$$

which is used in assessing stability of drawing process: in particular, value  $\sigma_i/\sigma_{T1.i} \leq 1$  indicates of a high probability of wire breaking at exit from the  $i$ -th die.

As a result, by setting the values  $m_i$  and  $R_{a.1.i}$  alternately and starting from the first pass, it is possible to determine the parameters that characterize the stability (or instability) of the multiple drawing process.

Technological calculations shall take into account the difference between the cumulative elongation of metal as per formula (6) and elongation  $\mu_i$  in the  $i$ -th pass as per expression (3) [1]:

$$\mu_{\Sigma i} = \prod_{i=1}^n \mu_i. \quad (15)$$

At  $\mu_i = \text{const}$  for all passes, equation (15) is presented as under – at exit from  $i$ -th pass:

$$\mu_{\Sigma i} = \mu_i^i, \quad (16)$$

– at the end of drawing path (at exit from the  $n$ -th pass):

$$\mu_{\Sigma n} = \mu_{\Sigma} = \mu_i^n. \quad (17)$$

The drawing multiplicity is calculated accordingly [1]:

$$n = \frac{\ln \mu_{\Sigma}}{\ln \mu_i}$$

or

$$\mu_i = \sqrt[n]{\mu_{\Sigma}} = \mu_{\Sigma}^{1/n}. \quad (18)$$

In practice the elongation factor in the first pass (at  $i = 1$ ) is assigned by 5–7% less than the average value with corresponding redistribution

of other  $\mu_i$  values within  $\mu_\Sigma$  framework. This leads to increase of lubricating layer thickness and improvement of friction conditions throughout the drawing process.

Also, it is known that one of the key factors in the efficiency of drawing process—that the overall metal elongation factor  $\mu_{\Sigma n}$  can be achieved by different number  $n$  of passes, which affects the other parameters of the process.

The disadvantage of the known approximation of statistical data [3] presented above is the lack of consideration of the obvious influence of the drawing ratio  $n$ , even at the same values of the overall degree of deformation  $\mu_\Sigma$ , and half the angle of the die cone on formation of friction mode and the respective drawing stability. This reduces their generalizing properties and prevents a reasonable determination of the conditions for the production process optimization.

The aim of the work was to develop approaches to the design of rational modes of multiple wire deformation with dry soap lubricant, taking into account the multiplicity and other parameters of the drawing process.

### 3. RESEARCH RESULTS

The presented shortcomings of the known researches have led to reasonability of expediency of returning to the analysis of initial statistical information [3] which is described above.

Now it has been taken into account that the lubricant attracting to the deformation zone during drawing depends on properties of the lubricant, diameter, roughness and yield strength of the wire rod, its movement speed, as well as conditions of metal deformation: die geometry and number of in-line deformation cycles (drawing multiplicity).

Therefore, when conducting an extended (as compared to work [3]) dimensional analysis, with appropriate algorithm [8], generalizing formula for  $i$ -th drawing pass was presented as under:

$$m_{1i} = X0V_i^{X1}\sigma_{T0i}^{X2}\alpha^{X3}R_{a0i}^{X4}d_0^{X5}d_{1i}^{X6}i^{X7}\rho_{1i}^{X8}, \quad (19)$$

where  $X0$ – $X7$  — arbitrary parameters, which further shall characterize the type of relationship of variables with the response function  $m_{1i}$ .

The last member  $\rho_{1i}$  was included in the list of variables in equation (19) to compensate for the dimension quantity ‘kg’, which is proper to variable  $\sigma_{T0i}$ , only and due to the impossibility of experimental determination of the dynamic viscosity of dry powdered lubricants before their entering to dies.

The dimensions of the indicated variables and the response functions are as under:  $V_i$ — $m \cdot s^{-1}$ ,  $\sigma_{T0i}$ — $N/m^2 = kg \cdot m^{-1} \cdot s^{-2}$ ,  $\alpha$ —dimensionless,  $R_{a0i}$ — $m$ ,  $d_0$ — $m$ ,  $d_{1i}$ — $m$ ,  $i$ —dimensionless,  $\rho_{1i}$ — $kg \cdot m^{-3}$ ,



$m_{1i} - \text{kg} \cdot \text{m}^{-2}$ .

In accordance with formula (19) equation «in dimensions of quantities» looks as follows:

$$\text{kg} \cdot \text{m}^{-2} = (\text{m} \cdot \text{s}^{-1})^{X1} (\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2})^{X2} 1^{X3} \text{m}^{X4} \text{m}^{X5} \text{m}^{X6} 1^{X7} (\text{kg} \cdot \text{m}^{-3})^{X8}. \quad (20)$$

Further, from expression (20) we obtained a system of equations that determine «balance» of quantity dimensions: for quantity dimension 'kg':  $1 = X2 + X8$ , for quantity dimension 'm':  $-2 = X1 - X2 + X4 + X5 + X6 - 3X8$ , for quantity dimension 's':  $0 = -X1 - 2X2$ .

Consideration of this system resulted in the following:

$$X8 = 1 - X2, X1 = -2X2, X4 + X5 + X6 = 1, \text{ or } X4 = 1 - X5 - X6.$$

This led to corresponding equivalent change in formula (20):

$$\text{kg} \cdot \text{m}^{-2} = (\text{m} \cdot \text{s}^{-1})^{-2X2} (\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2})^{X2} 1^{X3} \text{m}^{1-X5-X6} \text{m}^{X5} \text{m}^{X6} 1^{X7} (\text{kg} \cdot \text{m}^{-3})^{1-X2},$$

and when replacing the dimensions with appropriate parameters—identical replacement of equation (19) by the expression:

$$m_{1i} = X0 V_i^{-2X2} \sigma_{T0i}^{X2} \alpha^{X3} R_{a0i}^{1-X5-X6} d_0^{X5} d_{1i}^{X6} i^{X7} \rho_{1i}^{1-X2}. \quad (21)$$

After that, collecting parameters with the same dimensions into dimensionless groups, obtained:

$$\frac{m_{1i}}{\rho_{1i} R_{a0i}} = X0 \left( \frac{\sigma_{T0i}}{V_i^2 \rho_{1i}} \right)^{X2} \alpha^{X3} \left( \frac{d_0}{R_{a0i}} \right)^{X5} \left( \frac{d_{1i}}{R_{a0i}} \right)^{X6} (i)^{X7}. \quad (22)$$

The left part of formula (22) is similar to value  $\theta$  of friction mode as evidenced by its comparison with formulas (1) and (4).

The second member of the right part is clearly a characteristic of hydrodynamic phenomena, which largely accompany transfer of dry process lubricant to the deformation zone in the die.

According to the corresponding known theoretical formulas [4, 9], the thickness of the lubricating layer (amount of lubricant) on the wire is related inversely to the metal yield strength  $\sigma_{T0i}$ , and at the same time the drawing speed  $V_{0i}$  can cause both increase (more often) and decrease of the lubricating layer thickness due to a complex effect of temperature and the appearance of the so-called 'tunnel effect' [10].

The third member of formula (22) is dimensionless, so it can be represented both separately with its exponent  $X3$  and as a part of any dimensionless combination of this equation members.

It should be kept in mind that in mathematical expressions that describe the hydrodynamic lubricant transfer to the deformation zone

during metal processing by pressure, increasing half the angle of the die cone qualitatively determines the effect which is the same as parameter  $\sigma_{T0i}$  [4, 9].

The fourth and fifth members of the right-hand side of equation (22) together characterize both the relative height of the microrelief of the rod and the total metal elongation coefficient  $\mu_{\Sigma i}$  at the exit of  $i$ -th pass as per formula (6).

Since parameters X0–X8 at this stage are not quantified and taking into account the above considerations it becomes correct to convert expression (17) to the form:

$$M = \frac{m_{1i}}{\rho_{1i} R_{a,0,i}} = A \left( \frac{V_i^2 \rho_{1i}}{\sigma_{T0i} \alpha_i} \right)^{X1} \left( \frac{R_{a,0,i}}{d_{1i}} \right)^{X2} (\mu_{\Sigma i})^{X3} (i)^{X4}, \quad (23)$$

where  $A = \exp(X0)$  and values X0–X4, in general case, differ from the previous list of relevant parameters as in formula (19).

A polynomial was obtained by taking logarithm of the formula (23):

$$\ln \frac{m_{1i}}{\rho_{1i} R_{a,0,i}} = X0 + X1 \left( \frac{V_i^2 \rho_{1i}}{\sigma_{T0i} \alpha_i} \right) + X2 \ln \left( \frac{R_{a,0,i}}{d_{1i}} \right) + X3 \ln \mu + X4 \ln i. \quad (24)$$

Using the methods of regression analysis [11] in accordance with the array of experimental data presented above and expression (24), we determined the values of X0–X4 and A.

The adequacy of the model was judged by comparing the calculated (with the index ‘calc’) values of the dimensionless combination  $W = m_{1i}/(\rho_{1i} R_{a,0,i})$  as per formula (23) with its experimental (index ‘exp’) data using:

– levels of multiple correlation coefficient  $R$  and the average value of absolute (index ‘abs’) deviations for  $j$ -th implementation ( $1 \leq j \leq k$ ):

$G_{\text{abs}} = \sum_{j=1}^k |(W_{\text{exp}} - W_{\text{calc}}) / k|$  as generalizing characteristics of approximation quality;

– the value of the standard deviation  $S$  and average value (index ‘av’) of deviations with their signs:  $G_{\text{av}} = \sum_{j=1}^k (W_{\text{exp}} - W_{\text{calc}}) / k$  as characteristics of mutual deviation of the centre of scatter of experimental and calculated data.

At the first stage, the whole array of static data was subjected to consideration without selecting groups corresponding to the used lubricants and steels.

However, the obtained results were characterized by low values  $R \approx 0.5$ , and a detailed analysis of data revealed that too large values

$G_{\text{abs}}$ ,  $G_{\text{av}}$  and  $S$  took place in subgroups of used steels which were characterized by combination of all lubricants. This can be explained by the specific properties of lubricants (availability of additives, pre-heat treatment, *etc.*), which determine their efficiency and transfer to the deformation zone.

Therefore, in the next step, after extracting approximately 3% of the data that were recognized as errors, regression analysis was performed for the following options: all technological lubricants, when using all steels; process lubricant MP, with the use of which the corresponding steels were deformed; process lubricant MPT, with the use of which the corresponding steels were deformed; process lubricant MPL, with the use of which the corresponding steels were deformed.

The results and evaluation of the quality and approximation of experimental data as per formula (23) are presented in Table 1.

The analysis of the presented data testifies on satisfactory quality of the approximation, especially taking into account the fact that the output experimental data were obtained under production conditions, which are characterized by instability, insufficient controllability of parameters and negative human influence.

Therefore, it was considered that the obtained results can be used in the test mode to assess the optimality of the drawing technological process in relation to the rational determination of the number of passes in multiple deformation.

Calculation algorithm worked out by the authors includes the following steps.

1. Definition of initial conditions for metal deformation: diameter  $d_{\text{init}}$  of input wire-rod and its roughness value  $R_{\text{a,init}}$ ; desired final wire diameter  $d_n$ ; steel grade, kind of lubricant; drawing speed in the first  $V_{i=1}$  transition; value of dies semi-cone angle  $\alpha$ .

2. Determination of total elongation  $\mu_{\Sigma}$  as per formula (6) and desired number  $n = 1, 2, \dots, 6$  or 7 passes according to calculation variants, which will be analysed from view point of minimization  $n$  and possibility of drawing process under condition of no wire breaks.

3. Determination of elongation factors  $\mu_i$  in each of  $i$ -th passes  $1 \leq i \leq n$  in accordance with formula (18). In case of elongation factor reduction in the first pass (at  $i = 1$ ) less by 5–7% as against expected value, it will be necessary to revise accordingly other  $\mu_i$  within framework  $\mu_{\Sigma}$  for  $i > 1$  according to formula:

$$\mu_{i>1} = \sqrt[n-1]{\frac{\mu_{\Sigma}}{\mu_{i=1}}} = \left( \frac{\mu_{\Sigma}}{\mu_{i=1}} \right)^{\frac{1}{n-1}}. \quad (25)$$

**TABLE 1.** Parameters and quality characteristics of experimental data approximation.

Drawing conditions	Approximation parameters as per formula (23)					Approximation quality evaluation			
	$A = \exp X0$	$X1$	$X2$	$X3$	$X4$	$R$	$G_{abs}$	$G_{cep}$	$S$
All lubricants	$0.436 \cdot 10^{-5}$	-0.296	-1.065	1.688	-1.377	0.77	0.351	-0.172	0.423
MP	$10.930 \cdot 10^{-5}$	-0.258	-0.674	0.715	-0.058	0.86	0.511	-0.139	0.661
MPT	$4.493 \cdot 10^{-5}$	-0.411	-0.727	2.032	-1.534	0.82	0.3991	-0.222	0.4043
MPL	$5.385 \cdot 10^{-5}$	-0.098	-1.321	1.256	-1.535	0.78	0.29	-0.028	0.38

4. Calculation of cumulative elongation factors  $\mu_{\Sigma i}$  before exit from  $i$ -th passes as per formula (15).

5. In accordance with formula (6)—calculation of wire diameters values  $d_{1i}$  at exit from  $i$ -th dies as per expression:

$$d_{1i} = \frac{d_{init}}{\sqrt{\mu_{\Sigma i}}}. \tag{26}$$

6. Calculation of speed mode in direction of drawing path using formula [3]:

$$V_{1i} = \frac{\mu_{\Sigma i}}{\mu_{i=1}} V_{1i=1}, \tag{27}$$

where  $V_{1i=1}$ ,  $V_{1i}$ —wire speed at exit from the first and  $i$ -th die respectively.

It should be borne in mind the possible correction of the calculated values  $d_{1i}$ ,  $\mu_i$  and  $\mu_{\Sigma i}$ , that is due to the need to reconcile these parameters with the actual kinematics and necessity of some wire stock accumulation on intermediate traction drums of multiple drawing mills, as well as adopted at many metalwork companies dies diameter  $d_{1i}$  range (e.g. in production of the latter with a step of 0.05 mm).

7. Calculation of values  $\rho_{1i}$  for selected lubricant as per formula (7).

8. Calculation of values  $\sigma_{T0i}$  and  $\sigma_{T1i}$  for selected steel as per expression (12) and  $\sigma_{Tav.i}$ —as per formulas (10) or (11) for related passes.

9. Determination according to Table 1 data and formula (23) the level of dimensionless group  $M = m_{1i}/(\rho_{1i}R_{a.0.i})$  for  $i$ -th pass starting from the first one.

10. Determination according to equation (9) the level of dimensionless group  $\sigma_i/\sigma_{Tcep.i}$  for  $i$ -th pass starting from the first one. Thus, for the first iteration step considering level of friction factor it is necessary to adopt its probable average value  $f_i \approx 0.16$ .

11. For  $i$ -th pass starting from the first one, determination of dimensionless group  $r = R_{a.1.i}/R_{a.0.i}$  according to formula (8).

12. Determination of level of friction mode indicator  $\theta_i$  considering formulas (1), (5) and dimensionless groups  $M = m_{1i}/(\rho_{1i}R_{a.0.i})$  as per equation (23) at algorithm step 9 and  $r = R_{a.1.i}/R_{a.0.i}$  as per formula (8) at algorithm step 11 is carried out as a result of combination of corresponding dimensionless groups resulting in the following:

$$\theta = \frac{M}{r} \frac{R_{a.1.i}(\mu_i^{1/2} + 1)}{R_{a.0.i} + R_{a.1.i}}. \quad (28)$$

13. Calculation of adjusted value of friction factor  $f_i$  as per formula (13).

14. Comparison of  $f_i$  last value with its previous value. If difference is more than 5–10% you shall return to algorithm step 10, specifying the last calculated value  $f_i$  and, if necessary, repeat this iterative process till a satisfactory match of  $f_i$  last and previous values is obtained.

15. Use of required parameters for determination of ratio  $\sigma_i/\sigma_{T1i}$ , which characterizes safety margin of wire strength at exit from  $i$ -th die as per formula (14).

At  $\sigma_i/\sigma_{T1i} < 1$  it is necessary to pass to similar calculations for the subsequent drawing passes up to  $i = n$ . It should be kept in mind that the outlet geometric and kinematic parameters of the previous pass become inlet parameters for the next pass.

In case  $\sigma_i/\sigma_{T1i} \geq 1$  the calculation shall be terminated with conclusion that it is impossible to implement the drawing process with multiplicity  $n$  and selected distribution of elongation factors in passes. This necessitates an increase of  $n$ .

16. Repetition of actions according to the presented algorithm at higher drawing multiplicity  $n$ .

17. Formulation of a recommendation on the minimum acceptable drawing ratio, which will be optimal for certain conditions.

As an example, according to this algorithm calculated conditions of wire rod deformation with input diameter  $d_{\text{init}} = 6.5$  mm from steel Ст3кп 3 with initial yield strength  $\sigma_{T,\text{init}} = 275$  MPa and characteristics of strengthening  $u = 372$  MPa, value  $R_{a,\text{init}} = 3$   $\mu\text{m}$  on final diameter  $d_n = 2.45$  mm at a speed  $V_{i-1} = 1.5$  m/s of drawing at exit from the first die and value of half the angle  $\alpha = 0.105$  radian of die cone (step 1) with the use of lubricant MP, for which density  $\rho_{1i}$  is determined by equation (7).

Calculations were performed at multiplicity  $n = 4, 5, 6$  and 7 passes. The results obtained are shown in Table 2.

The analysis of these data shows that under the above conditions of dry wire drawing the rational (optimal) amount is  $n = 5$  passes of multiple deformation, which is consistent with the practice of sustainable

**TABLE 2.** The results of calculations by steps of the algorithm.

Steps of the algorithm															
Parameter defined using the appropriate formula, indicated by the number in parentheses: (...)															
Pass number $i$	$\mu_i, (3), (25)$	$\mu_{Si}, (6), (15)-(17)$	$d_{Li}, \text{mm} (26)$	$V_i, \text{M}/\text{c}, (27)$	$\rho_{Li}, \text{KR}/\text{M}^3, (7)$	$\sigma_{T0i}, \text{MIIa}, (12)$	$\sigma_{T1i}, \text{MIIa}, (12)$	$\sigma_{T, \text{av. } i}, \text{MIIa}, (10) \text{ or } (11)$	$\sigma_i / \sigma_{T, \text{av. } i}, (9)$	$r, (8)$	$\theta_i, (5)$	$f_i, (13) \text{ (after iteration)}$	$\sigma_i / \sigma_{T1i}, (14)$	Notes	
$n = 4$															
1	1.46	1.46	5.38	1.50	2821	275	724	574	0.18985	0.926	0.197	0.350	0.199	0.945	
2	1.69	2.47	4.14	2.54	3438	724	860	792	0.19317	1.147	0.229	0.362	0.197	1.156	Wire breaks
3	1.69	4.17	3.18	4.28	4556	860	1035	948	0.19760	1.142	0.231	0.369	0.196	1.154	Wire breaks
4	1.69	7.04	2.45	7.23	6175	1035	1262	1149	0.19847	1.140	0.231	0.371	0.195	1.154	Wire breaks
$n = 5$															
1	1.33	1.33	5.64	1.50	2764	275	704	561	0.18452	0.906	0.191	0.341	0.200	0.926	
2	1.52	2.02	4.58	2.28	3142	704	804	754	0.18798	0.964	0.201	0.348	0.199	0.977	
3	1.52	3.07	3.72	3.47	3841	804	927	866	0.19418	0.958	0.204	0.358	0.197	0.974	
4	1.52	4.67	3.02	5.27	4863	927	1079	1003	0.19778	0.954	0.206	0.364	0.196	0.972	
5	1.51	7.04	2.45	8.00	6175	1079	1262	1171	0.19740	0.952	0.205	0.364	0.196	0.973	
$n = 6$															
1	1.23	1.23	5.86	1.50	2729	275	688	550	0.18018	0.637	0.149	0.331	0.202	0.668	
2	1.42	1.75	4.92	2.13	2977	688	767	728	0.18267	0.889	0.187	0.337	0.201	0.906	

CONTINUATION OF TABLE 2.

3	1.42	2.48	4.14	2.99	3445	767	860	814	0.19088	0.883	0.191	0.351	0.199	0.902
4	1.42	3.52	3.47	4.25	4139	860	973	917	0.19541	0.879	0.194	0.359	0.197	0.900
5	1.42	5.00	2.91	6.03	5060	973	1107	1040	0.19786	0.876	0.195	0.363	0.197	0.899
6	1.41	7.04	2.45	8.49	6175	1107	1262	1185	0.19846	0.863	0.193	0.364	0.196	0.888
$n = 7$														
1	1.19	1.19	5.96	1.50	2718	275	681	546	0.17852	0.573	0.139	0.328	0.203	0.609
2	1.35	1.61	5.13	2.03	2898	681	747	714	0.18078	0.806	0.175	0.333	0.202	0.826
3	1.35	2.17	4.41	2.73	3239	747	823	785	0.18834	0.801	0.178	0.346	0.199	0.823
4	1.34	2.93	3.81	3.66	3747	823	912	868	0.19538	0.783	0.180	0.357	0.197	0.808
5	1.34	3.93	3.29	4.91	4404	912	1012	962	0.19926	0.780	0.181	0.364	0.196	0.806
6	1.34	5.26	2.84	6.57	5212	1012	1028	1020	0.20116	0.777	0.182	0.367	0.196	0.806
7	1.34	7.04	2.45	8.81	6175	1028	1262	1145	0.20126	0.777	0.182	0.367	0.196	0.805

industrial management of such a process. At  $n = 4$  passes, it is probable that the wire will break at the exit of the second die, thus preventing the process wire from entering the next die and in general—obtaining products. At  $n = 5, 6$  and  $7$ , the stability of the drawing process is ensured, but the use of extra cycles of metal deformation due to use in the process of the appropriate number of dies and traction drums with their electric drives, a priori reduces the energy efficiency of the technology.

#### 4. CONCLUSIONS

Using dimensional analysis, the generalized method of designing rational modes of multiple wire drawing with dry soap lubricants has been improved theoretically and experimentally, taking into account the friction mode in dies, other process parameters and determining the appropriate multiplicity of in-line metal deformation in the drawing mill.

The presented algorithm for calculating the rational number of passes of the rod through the dies during the in-line drawing allows in its practical implementation to achieve the stability of the technological process with no wire breaks and minimum required (optimal) number of cycles of metal deformation and corresponding energy consumption. The obtained results are consistent with the practice of industrial drawing with the use of dry soap-based lubricants and can be used in the development of new and testing the rationality of existing drawing processes.

In future, it is advisable to develop a program for automatic calculation of rational number of passes for dry drawing with application of the presented algorithm, as well as using the presented approach to expand the outlet statistical database taking into account properties of other steel grades that are deformed under dry and wet drawing processes.

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