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Theory of Spin Waves in an Easy-Plane Ferromagnetic Nanotube with a Spin-Polarized Current

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In the paper, dipole–exchange spin waves in a nanotube composed of an easy-plane ferromagnet in the presence of a spin-polarized current are studied theoretically. The magnetic dipole–dipole interaction, the exchange interaction, the magnetic anisotropy, the dissipation effects, and influence of the spin-polarized current are considered. For such spin waves, an equation for the magnetic potential is obtained and (for the case of longitudinal-radial waves) solved. As a result, the dispersion law for such waves is found. This dispersion law is complemented with the relation between the wave-vector components, which is shown to degenerate into a quasi-one-dimensional values' spectrum of the orthogonal wave-vector component nearly everywhere. As shown, in most cases, influences of the spin-wave dissipation and spin-polarized current on the real part of its frequency are negligible. Branches (which correspond to different orthogonal modes) of both the real and imaginary parts of the dispersion law are shown to be close to parabolic ones; distance between branches increases with increase of the mode number. Presence of the spin-polarized current can strengthen or weaken the spin-wave damping, creating the 'effective dissipation' and, in some cases, leading to a spin-wave generation. The condition of such a generation is found as well as limitations on the transverse modes' number, for which the generation is possible. As shown, for typical values of nanosystem parameters, only first several modes can be excited *via* such generation (in most cases, only zero mode or none). The method proposed in the paper can be applied to nanotubes (and other nanosystems) of more complex configurations.

Key words: spin wave, nanomagnetism, dipole–exchange wave, ferromagnet-

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У статті теоретично досліджено дипольно-обмінні спінові хвилі в нанотрубці з легкоплощинного феромагнетика за наявности спін-поляризованого струму. Враховано магнетну диполь-дипольну взаємодію, обмінну взаємодію, магнетну анізотропію, ефекти дисипації та вплив спін-поляризованого струму. Для зазначених спінових хвиль одержано та (для поздовжньо-радіяльних хвиль) розв'язано рівняння для магнетного потенціялу. Як результат, знайдено закон дисперсії таких хвиль. Цей закон дисперсії доповнено співвідношенням між компонентами хвильового вектора; показано, що це співвідношення майже всюди вироджується в квазиодновимірний спектер значень ортогональної компоненти хвильового вектора. Показано, що в більшості випадків впливи дисипації спінової хвилі та спін-поляризованого струму на дійсну частину її частоти є нехтовно малими. Показано, що гілки (які відповідають різним ортогональним модам) як дійсної, так і уявної частин закону дисперсії близькі до параболічних; віддаль між гілками збільшується зі збільшенням номера моди. Присутність спін-поляризованого струму може посилювати або послаблювати згасання спінової хвилі, створюючи «ефективну дисипацію» і в деяких випадках приводячи до ґенерації спінової хвилі. Знайдено умови такої ґенерації, а також обмеження на кількість поперечних мод, для яких ґенерація можлива. Показано, що для типових значень параметрів наносистеми за такої ґенерації можуть бути збуджені лише перші кілька мод (у більшості випадків лише нульова мода або жодної). Запропонований у статті метод може бути застосований до нанотрубок (та інших наносистем) більш складних конфіґурацій.

Ключові слова: спінова хвиля, наномагнетизм, дипольно-обмінна хвиля, феромагнетна нанотрубка, легкоплощинний феромагнетик, спін-поляризований струм.

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1. INTRODUCTION

Spin waves in magnetically ordered materials represent an actual and promising topic of research. In particular, spin waves in nanoscale systems are studied by a new sub-field of solid-state physics—magnonics [1]. Spin waves in nanosystems are promising for a variety of technical applications—both current and prospective. These applications concern mostly new devices for data storage, transfer, and processing [1–3] but are not limited to them. In particular, devices that use spin waves on nanoscale instead of electric currents—magnon devices—allow for faster, more efficient, and more reliable signal processing as well as computation on higher frequencies than current computer technology [1]. Spin waves can propagate through magnetic materials with minimal energy loss and can be easily manipulated using magnetic fields, electric fields, spin currents, or thermal gradients thus making them prospective for novel data transfer technologies [1]. These applications require precise theoretical models of excitation and propagation of spin waves in various nanosystems, so these models are extensively developed recently.

Properties of nanostructures—including spin-wave properties—are known to depend essentially on their size and shape. Therefore, spin waves are studied in different configurations of nanosystems individually. Synthesized recently magnetic nanotubes [4, 5] have found a wide range of technical applications—in particular, in magnetobiology. However, spin waves in nanotubes currently attract little attention. Known theoretical papers on the subject that are closest to the topic of the current paper are limited to investigating nanotubes composed of either isotropic or uniaxial easy-axis ferromagnets and does not account for spin wave damping (see, *e.g.*, [6]). Uniaxial easy-plane ferromagnets, however, possess a number of unique magnetic properties—in particular, due to a different degree of symmetry compared to similar systems composed of easy-axis ferromagnets. One should also note that effects associated with an energy dissipation can either significantly influence the pattern of spin waves in the system or be negligibly small (depending on the wave frequency, dimensions, shape and material of the system and other factors); see, *e.g.*, [7]. Nevertheless, dissipative spin waves in nanotubes composed of easy-plane ferromagnets currently remain poorly studied.

Magnetic nanostructures, in particular, magnetic nanotubes, can be used as waveguides for spin waves. Thus, a task of generating spin waves in these nanostructures becomes actual one. One of the ways of generating such waves [8] (usually in the microwave range) is using so-called spin-torque effect: change of the magnetization direction (switching or precession) in a thin layer of a ferromagnet when a spin-polarized current passes through it [8–10]. The influence of the spin-torque effect on the spin-wave pattern of the nanosystem can be either negligible or essential depending on the current density (see, *e.g.*, [9]) and, therefore, must be considered in a general case. Therefore, investigation of spin waves in magnetic nanotubes with a spin-polarized current and, in particular, investigation of generation of such waves via spin-torque effect represents a topical field of research.

The paper extends theoretical study of dipole–exchange spin waves in a circular nanotube composed of a uniaxial easy-plane ferromagnet started by the author in the previous paper [11]. The magnetic dipole–dipole interaction, the exchange interaction, the magnetic anisotropy, and the spin wave damping are considered. Unlike in the previous paper, spin-polarized current is assumed to pass through the tube. Such current is shown to change the spin wave characteristics and, in some cases, can lead to generation of a spin wave.

As a result, the dispersion law and the relation between the wave vector components for such waves are obtained and analysed. Effective dissipation, which can be either positive or negative depending on the direction and the density of the current, is shown to take place in the investigated nanosystem. The spin wave generation conditions are obtained.

2. THEORY

2.1. Problem Statement. Model Description

Let us consider a two-layer ferromagnetic nanotube with a circular cross-section and a spin-polarized current passing through it in the radial direction. We assume that one layer of the nanotube is 'fixed' in the sense of the magnetization direction, the second—'free', so the current passing through the 'free' layer (in the radial direction) becomes spin-polarized. Let us denote internal radius of the 'free' layer as a, and the external one as b and direct the Oz axis of the coordinate system along the symmetry axis of the tube (Fig. 1). The medium outside the tube is assumed non-magnetic.

Let us assume that the 'free' layer is composed of a uniaxial easy-plane ferromagnet with its anisotropy axis directed along the axis of symmetry of the nanotube (the vector $\boldsymbol{n}$ in Fig. 1), and the Oz axis of the co-ordinate system is also co-directed with $\boldsymbol{n}$. The ferromagnet parameters are denoted as follows: the exchange constant α, the uniaxial anisotropy parameter $\beta < 0$ (is considered constant), the gyromagnetic ratio γ (is considered constant). Let us consider the spin wave dissipation non-negligible and introduce the Gilbert damping constant of the

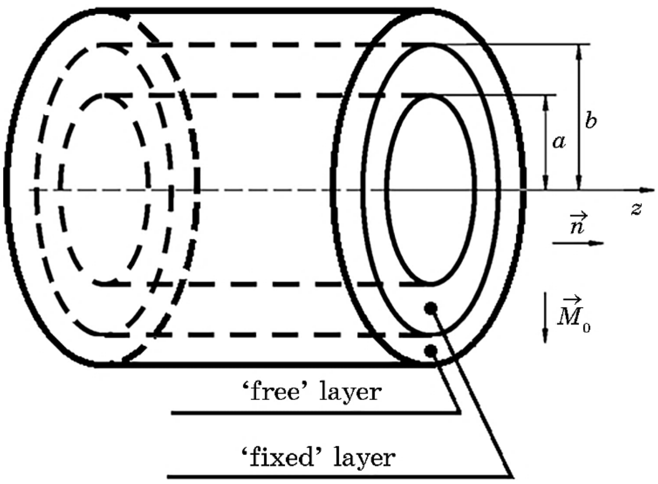

Fig. 1. Configuration of the investigated nanosystem.

ferromagnet α_G. The ground state magnetization $\boldsymbol{M}_0$ of the both layers are assumed to be directed radially (see Fig. 1) and have constant length in the entire volume of the 'free' layer. All components (in the cylindrical coordinate system) of the external magnetic field $\boldsymbol{H}^{(e)}$ are assumed stationary and homogeneous.

Let us consider a spin wave propagating in the 'free' layer of the above-described system along the axis Oz and take into account both the magnetic dipole–dipole and exchange interactions (as they both are essential for a nanoscale system) as well as the anisotropy, damping and spin-torque effect in the Landau–Lifshitz equation. The wave is assumed to be linear so the magnetization $\boldsymbol{m}$ and the magnetic field $\boldsymbol{h}$ of the wave are small perturbations of the overall magnetization $\boldsymbol{M}$ and the internal magnetic field (inside the 'free' layer) $\boldsymbol{H}^{(i)}$, correspondingly: $\boldsymbol{M}=\boldsymbol{M}_0+\boldsymbol{m}$, $\boldsymbol{H}^{(i)}=\boldsymbol{H}_0^{(i)}+\boldsymbol{h}$, where $\boldsymbol{H}_0^{(i)}$ is the ground state internal magnetic field. Thus, the inequalities $|\boldsymbol{m}|<<|\boldsymbol{M}_0|$, $|\boldsymbol{h}|<<|\boldsymbol{H}_0^{(i)}|$ fulfil. The task of the paper is to obtain the dispersion relation for such wave, relation between the wave vector components and determine the condition of a spin wave generation in the 'free' layer.

2.2. Starting Relations

Let us introduce the cylindrical coordinate system (ρ, θ, z). For the 'free' layer, $\boldsymbol{M}_0=M_0\boldsymbol{e}_\rho$, $M_0=\text{const}$, where $\boldsymbol{e}_\rho$ is a unit vector for the coordinate ρ. Then, a linearized Landau–Lifshitz equation for the considered nanotube in the absence of the spin-polarized current can be written analogously to [11] as follows:

$$\frac{\partial \boldsymbol{m}}{\partial t} = \gamma\left(M_0\boldsymbol{e}_\rho \times \left(\boldsymbol{h} + \alpha\Delta\boldsymbol{m} + \beta m_z\boldsymbol{e}_z - \frac{H_0^{(i)}}{M_0}\boldsymbol{m} + \frac{\alpha_G}{\gamma M_0}\frac{\partial \boldsymbol{m}}{\partial t}\right)\right), \tag{1}$$

where $\boldsymbol{e}_z$ is a unit vector for the coordinate z.

In order to take into account, the spin-polarized current, let us use the Slonczewski–Berger spin-transfer term analogously to the previous papers of the author [12, 13]. Namely, we assume that the 'free' nanotube layer is thin enough to use the form of the term obtained for a flat film [9]. This term in the linearized form after taking into account $\boldsymbol{M}_0||\boldsymbol{e}_z$, $\boldsymbol{m}\perp\boldsymbol{e}_z$ can be written for the investigated nanotube as follows:

$$\boldsymbol{t}_s = \frac{\varepsilon\gamma\hbar J}{2eM_0^2(b-a)}[\boldsymbol{M}_0 \times [\boldsymbol{m} \times \boldsymbol{e}_\rho]], \tag{2}$$

where ε is the dimensionless spin-polarization efficiency, J is the electric current density (is considered constant) and e is the modulus of the electron charge.

For the perturbations of the magnetization and magnetic field in a form of the travelling waves $\boldsymbol{m} = \boldsymbol{m}_0(\boldsymbol{r})\exp(i\omega t) = \boldsymbol{m}_{\perp 0}(\rho,\theta)\exp(i\omega t - ik_{\|z})$, $\boldsymbol{h} = \boldsymbol{h}_0(\boldsymbol{r})\exp(i\omega t) = \boldsymbol{h}_{\perp 0}(\rho,\theta)\exp(i\omega t - ik_{\|z})$ (where ω is the spin wave frequency and $k_{\|}$ is the longitudinal wave number), Eq. (1) with the spin-transfer term (2) can be rewritten as follows:

$$i\omega \boldsymbol{m}_0 - \frac{\gamma\varepsilon\hbar J \boldsymbol{m}_0}{2eM_0(b-a)} = \gamma\left(M_0\boldsymbol{e}_\rho \times \left(\boldsymbol{h}_0 + \alpha\Delta\boldsymbol{m}_0 + \beta m_{0z}\boldsymbol{e}_z + \left(\frac{i\omega\alpha_G}{\gamma} - H_0^{(i)}\right)\frac{\boldsymbol{m}_0}{M_0}\right)\right). \tag{3}$$

Analogously to the previous paper [11], let us note that this Landau–Lifshitz equation combined with the Maxwell equation $\mathrm{div}\boldsymbol{h} = -4\pi\mathrm{div}\boldsymbol{m}$ forms a system of equations in which the magnetization perturbation vector can be eliminated. Let us use the magnetostatic approximation (that can be applied for typical spin waves; see, *e.g.*, [14]) and introduce the magnetic potential $\Phi(\boldsymbol{r},t) = \Phi_0(\boldsymbol{r})\exp(i\omega t) = \Phi_{\perp 0}(\rho,\theta)\exp(i\omega t - ik_{\|z})$; so, for the magnetic field, the relations $\boldsymbol{h} = \nabla\Phi$, $\boldsymbol{h}_0 = \nabla\Phi_0$ fulfil. Then, after the above-mentioned elimination of the magnetization perturbation, one can obtain the following equation for the magnetic potential:

$$\left(\frac{\left(\omega + \frac{i\gamma\varepsilon\hbar J}{2eM_0(b-a)}\right)^2}{\gamma^2 M_0^2} - \left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + \frac{i\omega\alpha_G}{\gamma M_0}\right)\left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + \frac{i\omega\alpha_G}{\gamma M_0} - 4\pi\right)\right)\Delta\Phi_0 -$$

$$-4\pi\left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + \frac{i\omega\alpha_G}{\gamma M_0}\right)\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi_0}{\partial\rho}\right) + \frac{\left(\omega + \frac{i\gamma\varepsilon\hbar J}{2eM_0(b-a)}\right)\beta k_{\|}}{\gamma M_0 \rho}\frac{\partial\Phi_0}{\partial\theta} +$$

$$+\beta k_{\|}^2\left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + \frac{i\omega\alpha_G}{\gamma M_0}\right)\Phi_0 = 0. \tag{4}$$

Let us note that as the investigated nanotube is considered thin ($(b-a)/a \ll 1$), for the internal magnetic field the relation for a flat film fulfils approximately: $\boldsymbol{H}_0^{(i)} \approx \boldsymbol{H}^{(e)} - 4\pi\boldsymbol{M}_0$. In particular, if the external magnetic field is directed along the tube axis analogously to the previous papers by the author [12, 15], which consider nanotubes made of an easy-axis ferromagnet, one can obtain

$$H_0^{(i)} \approx \sqrt{\left(H^{(e)}\right)^2 + \left(4\pi M_0\right)^2}\,.$$

The spin-polarized current does not change the magnetic field as it is directed radially.

3. RESULTS AND DISCUSSION

3.1. Dispersion Relation

Unlike the case of nanotubes composed of an easy-axis ferromagnet (including previous papers of the author [12, 15]), for the case of an easy-plane ferromagnet, it is not possible (in general case) to seek a solution of Eq. (4) in the form of a linear combination of cylindrical functions because of the presence of two derivatives $\partial\Phi_0/\partial\theta$, $(1/\rho)\partial/\partial\rho(\rho\partial\Phi_0/\partial\rho)$. However, it becomes possible if angular oscillations are absent. Therefore, let us consider the particular case of longitudinal-radial waves for which, in particular, the relation $\partial\Phi_0/\partial\theta = 0$ fulfils.

In such case, a solution of Eq. (4) can be sought as follows:

$$\Phi_0 = \left(A_1 J_0(k_\perp\rho) + A_2 N_0(k_\perp\rho)\right)\exp(-ik_\parallel z)\,; \tag{5}$$

Here, A_1 and A_2 are constants, J_n and N_n are the Bessel and Neumann functions of order n, correspondingly, $k_\perp$ is the transverse wave number. In the considered case, n is the transverse-angular oscillatory mode number, which can only be equal to 0 (zero transverse-angular mode). After substituting the solution (5) into Eq. (4), one can obtain the following dispersion relation:

$$\frac{\omega}{\gamma M_0} = \frac{1}{1+\alpha_G^2}\left(i\left(\frac{\kappa\left(1+\alpha_G^2\right)}{\gamma M_0} + \frac{\alpha_G}{2}\left(2K^2+R^2\right)\right) \pm \right.$$
$$\left. \pm\sqrt{\left(1+\alpha_G^2\right)\left(K^2\left(K^2+R^2\right)+\frac{\kappa^2\left(1+\alpha_G^2\right)}{\gamma^2 M_0^2}\right) - \left(\frac{\kappa\left(1+\alpha_G^2\right)}{2\gamma M_0} + \frac{\alpha_G}{4}\left(2K^2+R^2\right)\right)^2}\,\right), \tag{6}$$

where

$$K^2 = \alpha k^2 + \frac{H_0^{(i)}}{M_0} \approx \alpha k^2 + \sqrt{\left(\frac{H^{(e)}}{M_0}\right)^2 + 16\pi^2},\quad R^2 = \left(4\pi + |\beta|\right)\frac{k_\parallel^2}{k^2},$$
$$\kappa = \frac{\gamma\varepsilon\hbar J}{2eM_0(b-a)}, \tag{7}$$

and $k = \sqrt{k_\parallel^2 + k_\perp^2}$ is the total wavenumber.

After taking into account the fact that spin waves can only be excited, when the damping parameter is small (α_G has the order of magnitude 0.1 or less), this dispersion relation can be simplified as follows (the root with the positive real part):

$$\omega \approx \gamma M_0 \left(i\left(\frac{\kappa}{\gamma M_0} + \alpha_G \left(K^2 + \frac{R^2}{2} \right) \right) + \sqrt{K^2 \left(K^2 + R^2 \right) - \frac{2\kappa\alpha_G}{\gamma M_0} \left(K^2 + \frac{R^2}{2} \right)} \right). \tag{8}$$

Eq. (8) represents the sought dependence of the spin wave frequency on the wave vector components.

3.2. Relation between the Wave Vector Components

Let us note that the above-obtained dispersion law (8) contains dependence on two components of the wave vector (transverse and longitudinal). Then, for more complete specification of the spin-wave pattern, this law must be supplemented by either a spectrum of values of at least one of these components or a relation between them.

Analogously to the previous paper [11], let us use standard boundary conditions for the magnetic field $b_{1n} = b_{2n}$, $h_{1\tau} = h_{2\tau}$ (here, $\boldsymbol{b}$ is the wave magnetic induction vector, medium 1 is the investigated ferromagnet, 2 is the external medium, n means the normal, and τ is the tangential to the interface components of the vector). In the process of applying these boundary conditions on both interfaces of the 'free' layer, and, thus, bounding the magnetic potential inside and outside the 'free' layer, one can notice that the situation is physically different from the one investigated in the previous paper. Namely, while in [11] the external medium (outside the tube) was assumed to be non-magnetic, in the current research outside material that corresponds to the 'fixed' layer is ferromagnetic (the inner medium, however, remains non-magnetic). Nevertheless, this fact does not change corresponding mathematical transformations because this layer is 'fixed' and does not sustain spin waves.

As a result, one can use expressions for the relation between the wave vector components used in [11] for the spin waves investigated in the current paper, namely, relations for the case of a thin tube $(b-a)/a << 1$ as the current research is limited to this case:

$$\mathrm{tg}\left(k_\perp (b-a)\right) = \frac{\dfrac{k_\parallel}{k_\perp}\left(\dfrac{I_0'(k_\parallel a)}{I_0(k_\parallel a)} - \dfrac{K_0'(k_\parallel b)}{K_0(k_\parallel b)} \right)}{1 + \left(\dfrac{1}{2k_\perp a} + \dfrac{k_\parallel}{k_\perp} \dfrac{I_0'(k_\parallel a)}{I_0(k_\parallel a)} \right)\left(\dfrac{1}{2k_\perp b} + \dfrac{k_\parallel}{k_\perp} \dfrac{K_0'(k_\parallel b)}{K_0(k_\parallel b)} \right)}, \tag{9}$$

where I_0, K_0 are the modified Bessel and Neumann functions of order 0, correspondingly. Analogously to [11], this relation degenerates to a quasi-one-dimensional spectrum for the transverse wavenumber

$$k_\perp = \pi s/(b-a), \tag{10}$$

(where s is an integer, number of the transverse mode) on the most part of the ranges of values of the components $k_{\parallel}$ and $k_{\perp}$. (The component $k_{\parallel}$ has the order of magnitude of the reciprocal nanotube length or more and the component $k_{\perp}$ has the order of magnitude of the reciprocal nanotube thickness. Therefore, the relations $k_{\perp}a >> 1$, $k_{\perp}b >> 1$, $k_{\parallel} << k_{\perp}$ fulfil on the most part of the ranges of values of the components $k_{\parallel}$ and $k_{\perp}$. For $k_{\perp}a >> 1$, $k_{\perp}b >> 1$ and either $k_{\parallel} << k_{\perp}$ or $k_{\parallel} >> k_{\perp}$, the relation between the wave vector components can be approximately written in a quasi-one-dimensional form (10) [11]. The strongest deviation from this simplified relation is observed when $k_{\parallel}$ and $k_{\perp}$ have close values).

3.3. Discussion

Therefore, the sought dispersion relation for the investigated spin waves can be written in the form (8) with the parameters (7) and $k_2 = k_{\parallel 2} + k_{\perp 2}$. The longitudinal wave-vector component $k_{\parallel}$ can be considered to change continuously while the orthogonal wave-vector component $k_{\perp}$ is defined by the implicit relation (9). This relation can be reduced to the quasi-one-dimensional spectrum (10) for $k_{\perp}$ nearly everywhere. Let us analyse the obtained results.

First, let us note that the relation between the wave vector components—in either of the forms (9) or (10)—is similar to the analogous relations obtained for a nanotube composed either of an easy-plane [11] or an easy-axis (after limiting the mode number n to 0) [15] ferromagnet in the absence of the spin-polarized current. Graphical representation of the relation (9) can be seen in Fig. 2.

Analogously to the previous paper [11], one can see from Fig. 3 that the relation between the wave vector components, really, is close to the

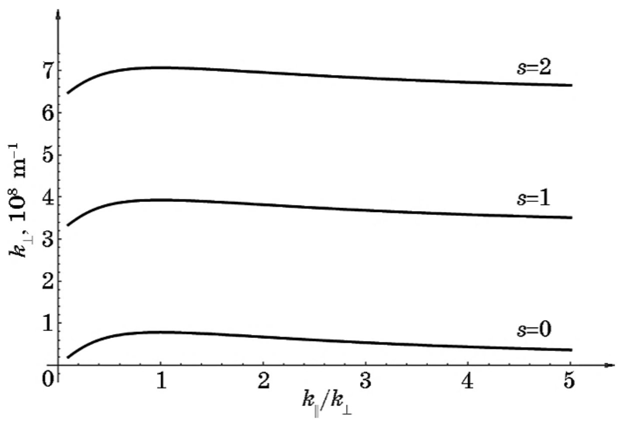

Fig. 2. Dependence of $k_{\perp}$ on $k_{\parallel}/k_{\perp}$ for the investigated composite nanotube with the radii of the 'free' layer $a = 50$ nm, $b = 60$ nm.

quasi-one-dimensional spectrum of values of the component $k_\perp$ (10). The strongest deviation from this simplified relation is observed when $k_\parallel$ and $k_\perp$ have close values, which corresponds to general considerations. (Namely, mutual influence of the longitudinal spin waves and the transverse spin excitations should be the strongest when their wavenumbers have close values.)

Then, let us analyse the real part of the frequency. It becomes approximately equal to the non-dissipative expression in the absence of a spin-polarized current obtained by the author in earlier paper [16] when the condition

$$|\kappa| << \kappa_{cr1} = \gamma M_0 K^2 \left(K^2 + R^2\right) \Big/ \left[2\alpha_G \left(K^2 + \frac{R^2}{2} \right) \right] \tag{11}$$

fulfils. The radical in the expression (8) vanishes and then becomes imaginary for the positive values of the parameter κ when passes the first critical value κ_{cr1}. Therefore, the entire spin wave vanishes: as the 'effective dissipation' is positive for $\kappa > 0$, the spin wave cannot be excited for $\kappa \geq \kappa_{cr1}$ for the '–' root of the frequency determined by the '±' sign in (6). The same applies for the '–' root (because imaginary part of the frequency also remains positive). For the negative values of κ, the radical in (8) remains real everywhere. Let us check whether this vanishing of the spin wave can be achieved for typical values of the nanosystem parameters.

Let us choose the following values for the 'free' layer ferromagnet: $\beta = -1$, $\alpha = 10^{-12}$ cm^{-2}, $\gamma = 10^7$ Hz/Gs, $M_0 = 10^3$ Gs (typical values for ferromagnets used in synthesized recently nanosystems; see, *e.g.*, [17, 18]). The Gilbert damping constant α_G for a typical ferromagnetic nanosystem used in experiments with a spin-polarized current can be chosen in the range of approximately 0.02–0.2; see, *e.g.*, [19, 20]. Thickness of a typical nanotube varies from unities to tens of nm. Then, for $b - a = 10$ nm in the absence of the external magnetic field and transverse spin excitations, the first critical value κ_{cr1} has the order of magnitude 10^{12}–10^{13} Hz depending on the value of α_G. Therefore, the corresponding critical value of the current density $J_{cr}^{(1)}$ has the order of 10^{20}–10^{22} Fr/(s·cm^2) ($3 \cdot 10^{10}$–$3 \cdot 10^{12}$ A/cm^2). This value exceeds essentially typical current densities in the corresponding experiments (see, *e.g.*, [10]). Thus, for typical values of the investigated nanosystem parameters, the condition (11) fulfils, and the real part of the frequency is approximately equal to that for a non-dissipative system in the absence of a spin-polarized current:

$$\mathrm{Re}\,\omega \approx \gamma M_0 K \sqrt{K^2 + R^2} = \gamma M_0 \sqrt{\left(\alpha k^2 + \frac{H_0^{(i)}}{M_0}\right)\left(\alpha k^2 + \frac{H_0^{(i)}}{M_0} + \left(4\pi + |\beta|\right)\frac{k_\parallel^2}{k^2}\right)}. \tag{12}$$

In particular, the above-described vanishing of the real part of the frequency does not happen. Even for low values $M_0 \cong 10^2$ Gs, $b-a \cong 1$ nm and high values $\alpha_G \cong 0.2$, $J \cong 10^8$ A/cm^2 difference between the exact expression given by (8) and approximate expression (12) can be considered negligible, albeit noticeable (see Fig. 3.).

Dependence of the real part of the spin-wave frequency given by the simplified relation (12) on $k_\parallel$—analogous to the one obtained in [11]—for the above-mentioned typical values of the nanotube parameters (specifically, $b-a=10$ nm) and with the spectrum of the transverse wave-vector component in the form (10) is represented in Fig. 4. As one can see, branches (that correspond to different orthogonal mode number s) of the dependence are close to parabolic. Distance between them increases with increase of the number s.

Now, let us analyse the imaginary part of the dispersion law (8):

$$\mathrm{Im}\,\omega = \kappa + \gamma M_0 \alpha_G \left(K^2 + \frac{R^2}{2} \right) = \kappa + \gamma M_0 \alpha_G \left(\alpha k^2 + \frac{H_0^{(i)}}{M_0} + \frac{\left(4\pi + |\beta|\right) k_\parallel^2}{2k^2} \right). \quad (13)$$

Unlike the real part of the spin wave frequency, the imaginary part in general case is affected essentially by the spin-polarized current. The imaginary part contains a positive (damping) addend that describes the spin wave dissipation and an addend that describes the effects of the spin-polarized current; the latter can be positive or negative depending on the direction of the current. (Positive values of κ correspond to the current passing from the 'fixed' nanotube layer to

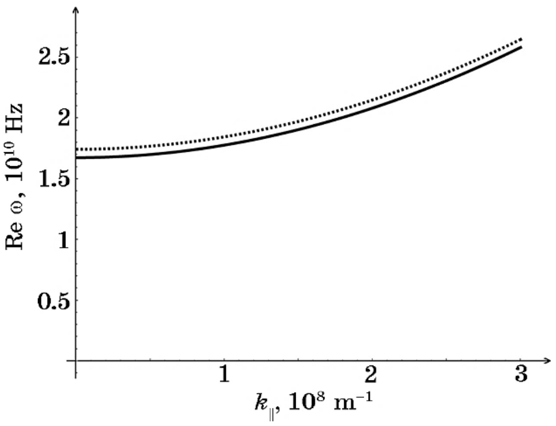

Fig. 3. Dependence of Reω on $k_\parallel$ for the investigated spin waves according to exact (solid line) and approximate (dashed line) dispersion laws for zero transverse mode ($s=0$). The following values of the 'free' layer parameters are used: thickness $b-a=1$ nm, external magnetic field $H^{(e)}=0$ and the ferromagnet parameters $\alpha = 10$–12 cm^{-2}, $\beta=-1$, $\gamma=10^7$ Gs/Hz, $M_0=10^2$ Gs.

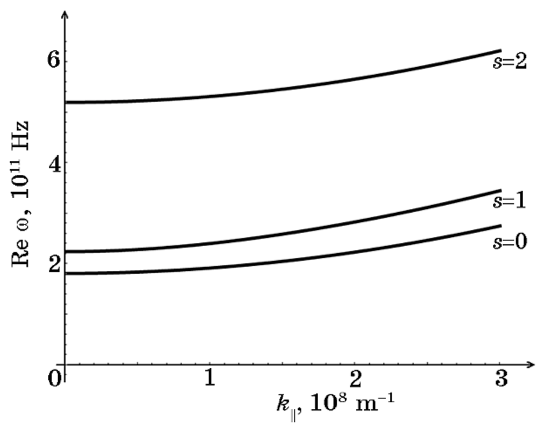

Fig. 4. Dependence of Reω on $k_{\|}$ for the investigated spin waves with the following values of the 'free' layer parameters: thickness $b-a=10$ nm, external magnetic field $H^{(e)}=0$ and the ferromagnet parameters $\alpha=10^{-12}$ cm^{-2}, $\beta=-1$, $\gamma=10^{7}$ Gs/Hz, $M_0=10^{3}$ Gs.

the 'free' one, negative—*vice versa*.) As a result, imaginary part of the frequency can be positive or negative ('effective dissipation')—and, therefore, the excitation or damping process can dominate, so that the spin wave can grow or attenuate in amplitude with time, correspondingly. In the first case, generation of a spin wave takes place.

If the parameter κ and, therefore, the current density J, are positive, the presence of the spin-polarized current increases the damping (the sign of the spin-torque addend in (13) is the same as that of the damping addend). For negative values of J, the sign of the damping and spin-torque addends in (13) are opposite. Therefore, spin-torque effect partially compensates the dissipation ('effective dissipation' is weaker than the dissipation in the absence of the spin-polarized current). When the parameter κ reaches the second critical value,

$$\kappa_{cr2}=-\gamma M_0\alpha_G\left(\alpha k^2+\frac{H_0^{(i)}}{M_0}+\frac{1}{2}\left(4\pi+|\beta|\right)\frac{k_{\|}^2}{k^2}\right), \quad (14)$$

the 'effective dissipation' vanishes. In this case, the spin wave does not grow or attenuate with time: a self-sustained magnetization precession occurs.

If the parameter κ is less than κ_{cr2}, the 'effective dissipation' becomes negative. Effectively, that means that the spin wave and, in particular, small spin-wave fluctuations, which occur spontaneously, grow with time exponentially $\boldsymbol{m}\propto\exp(\omega t)$ until the linear model limitations are reached: generation of a spin wave takes place.

Critical value of the current density J_{cr2} that correspond to κ_{cr2} for the above-mentioned typical values of the 'free' layer parameters ($\beta = -1$, $\alpha = 10^{-12}\,\text{cm}^{-2}$, $\gamma = 10^7$ Hz/Gs, $M_0 = 10^3$ Gs, $b - a = 10$ nm, $\alpha_G = 0.05$) has the order of 10^{18} Fr/(s·cm^2) ($3\cdot 10^8$ A/cm^2) for the zero transverse mode $s = 0$ (which is near the highest values of spin-polarized spin current densities used in experiments and exceeds them) and more for higher modes. On the other hand, for low enough values of M_0 and $b - a$ and high enough value of α_G, critical value of the current density lies within the admissible range, at least, for the zero transverse mode. *E.g.*, for $M_0 = 3\cdot 10^2$ Gs, $b - a = 3$ nm, $\alpha_G \cong 0.02$ (with the rest of the parameters remaining the same), J_{cr2} for $s = 0$ has the order of $3\cdot 10^6$ A/cm^2 that is easily achievable. For the first ($s = 1$) or higher modes, J_{cr2} exceeds current densities in the corresponding experiments unless the exchange constant is small enough (*e.g.*, for $\alpha = 2.5\cdot 10^{-12}\,\text{cm}^{-2}$, the first mode $s = 1$ can be excited, but not higher ones). For $M_0 = 10^2$ Gs, $b - a = 3$ nm, $\alpha_G = 0.02$, $\alpha = 2.5\cdot 10^{-13}\,\text{cm}^{-2}$ (with rest of the parameters remaining the same), 5 modes ($s = 0$–4) exist within the admissible range of current densities.

Considering the fact that minimal value of the component $k_\parallel$ has the order of magnitude of the reciprocal nanotube length and, therefore, $k_\parallel^{\min} << k_\perp$ (for $s > 0$), $\alpha(k_\parallel^{\min})^2 << 1$, for an experiment with the maximal current density $J_{\max}$ generation condition for zero transverse mode can be fulfilled, if

$$\frac{\alpha_G M_0^2 (b-a)\left(6\pi + |\beta|\right) e}{\varepsilon \hbar J_{\max}} \le 1, \tag{15}$$

and for the s-th mode ($s > 0$),

$$\frac{2e\alpha_G M_0^2 (b-a)}{\varepsilon \hbar J_{\max}} \left(4\pi + \alpha \left(\frac{\pi s}{b-a} \right)^2 \right) \le 1. \tag{16}$$

Dependence of the imaginary part of the spin-wave frequency given by the relation (13) on $k_\parallel$ and J for two sets of values of the nanotube parameters (one of which is the same as used for Fig. 4 and other for the values that allow the spin-wave generation) with the spectrum of the transverse wave vector component in the form (10) is represented in Fig. 5. As one can see, depending on values of the nanosystem parameters, spin-polarized current really can either contribute 'effective dissipation' weakly (Fig. 5, *a*) or essentially, leading to spin-wave generation for some values of the current density (Fig. 5, *b*). Branches of the dependence that correspond to different transverse modes (defined by number s) display regularities analogical to those observed for the dependence $\text{Re}\,\omega(k_\parallel)$. Namely, they are close to parabolic (for fixed value of J), unlike for $\text{Re}\,\omega$, the dependence is close to linear, and distance between them increases with increase of the number s.

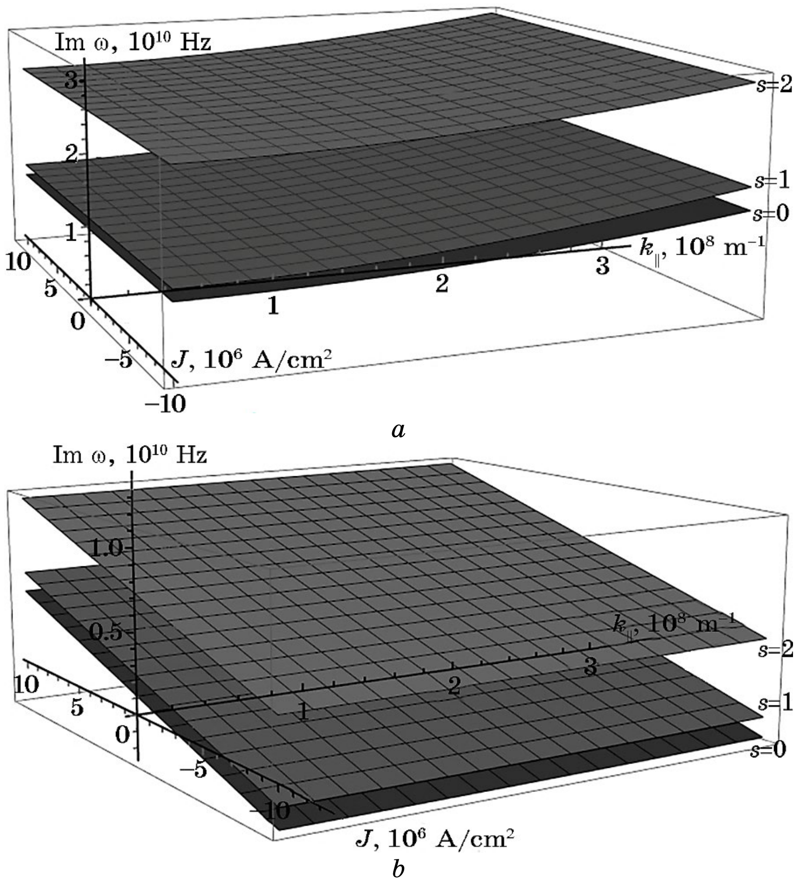

Fig. 5. Dependence of Imω on $k_{\|}$ and J for the investigated spin waves with the following values of the 'free' layer parameters: $H^{(e)}=0$, $\beta=-1$, $\gamma=10^7$ Gs/Hz and *a*) $b-a=10$ nm, $\alpha=10^{-12}$ cm^{-2}, $M_0=10^3$ Gs, $\alpha_G=0.05$, *b*) $b-a=3$ nm, $\alpha=2.5\cdot10^{-13}$ cm^{-2}, $M_0=3\cdot10^2$ Gs, $\alpha_G=0.02$.

4. CONCLUSION

Thus, dissipative dipole–exchange spin waves in a circular two-layer ferromagnetic nanotube with a spin-polarized current have been studied theoretically in the paper. One of the nanotube layers is considered 'fixed' in the sense of the magnetization orientation, the other (in which the investigated spin waves travel)—'free'. The 'free' layer is assumed composed of an easy-plane uniaxial ferromagnet with the magnetic anisotropy axis directed along the tube axis. The spin-polarized current passes through the 'free' layer in the direction orthogonal to its surface. The magnetic dipole–dipole interaction, the exchange interaction, the magnetic anisotropy, the spin wave dissipation and (unlike in the previous paper) influence of the spin-polarized current are taken into account.

For the above-described spin waves, an equation for the magnetic potential has been obtained and (for the case of longitudinal-radial waves) solved. As a result, the dispersion law for the investigated waves has been found. This dispersion law has been complemented with the relation between the wave-vector components, which has been shown to degenerate into a quasi-one-dimensional values' spectrum of the orthogonal wave vector component nearly everywhere.

For the obtained spectral characteristics of the investigated spin waves, graphical representations have been given and numerical evaluations have been per-formed. It has been shown that, in most cases, influences of the spin-wave dissipation and spin-polarized current on the real part of the dispersion law—dependence of the real part of the frequency on the longitudinal wave-vector component—are negligible. Branches of this dependence (that correspond to different orthogonal modes) are shown to be close to parabolic; distance between them increases with increase of the mode number.

For the imaginary part of the frequency (that describes the spin-waves dissipation), influence of the spin-polarized current can be essential or negligible depending on the values of the nanosystem parameters. Branches of the dependence of the imaginary part of the frequency on the longitudinal wave vector component and the current density are also close to parabolic (for fixed value of the current density)—but unlike the dependence for the real part of the frequency, the dependence is close to linear—and distance between also them increases with increase of the number s.

It has been shown that, in general case, presence of the spin-polarized current can strengthen or weaken the spin-wave damping, creating the 'effective dissipation'. When the 'effective dissipation' becomes negative, the wave grows in amplitude with time, thus leading to a spin wave generation. The condition of such a generation has been found as well as limitations on the transverse-modes' number, for which the generation is possible. It has been shown that, for typical values of the nanosystem parameters, only first several modes can be excited *via* such a generation: in most cases, only zero mode or none.

The method proposed in the paper can be applied to nanotubes of more complex configurations—for instance, with an elliptic cross-section—as well as for more complex configurations of tube-type nanosystems in general.

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