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Current-Driven Depinning of Elastic Vortex Filaments in Superconductors with Columnar Defects

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In the present work, the problem of an elastic Abrikosov-vortex filament depinning from a columnar defect in a 3D-anisotropic superconductor plate is considered within the framework of Lagrangian classical mechanics. We consider the problem of vortex depinning from columnar defect in a rather thick superconducting plate with thickness $d > 2\lambda$ (λ is the London penetration depth) under the action of transport Meissner current with a density $j(r, t)$ flowing in the surface screening layer of width $\cong \lambda$. The conditions for the occurrence of instability of the pinned state of the vortex are investigated and the corresponding depinning critical-current density on the specimen surface, at which the vortex filament starts its escape from the columnar defect, is calculated. The time for the vortex-depinning process and its dependences on both the sample thickness and the transport-current value are calculated. The dependence of the average critical-current density on the plate thickness d is also found.

Key words: superconductor, Abrikosov vortex, flux quantum, coherence length, vortex pinning, columnar defect, critical current.

У даній роботі в рамках Лягранжової класичної механіки розглянуто

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задачу про депінінг пружної вихрової нитки Абрикосова від стовпчастого дефекту в пластині 3D-анізотропного надпровідника. Розглянуто задачу про депінінг вихору зі стовпчастого дефекту в достатньо товстій надпровідній пластині товщиною $d > 2\lambda$ (де λ — Лондонова глибина проникнення) під дією транспортного Майсснерового струму з густиною $j(r, t)$, що протікає у поверхневому екранівному шарі шириною $\cong \lambda$. Досліджено умови виникнення нестабільності закріпленого стану вихору та розраховано відповідну густину критичного струму депінінгу на поверхні зразка, за якої вихрова нитка починає вихід зі стовпчастого дефекту. Розраховано час процесу депінінгу вихору та його залежність від товщини зразка і значення транспортного струму. Також знайдено залежність середньої критичної густини струму від товщини пластини d .

Ключові слова: надпровідник, вихор Абрикосова, квант потоку, довжина когерентності, пінінг вихору, стовпчастий дефект, критичний струм.

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1. INTRODUCTION

Modern electrical engineering requires materials with high current-carrying capacity. Such materials can be high-temperature superconductors (HTS) capable of conducting non-dissipative current with a density of the order 10^6 A/cm² at liquid nitrogen temperatures ($T \leq 78$ K) [1, 2]. The current-carrying capacity of superconducting materials usually is characterized by the critical current density—the maximal density value of the non-dissipative current, j_c . Today, the value of j_c (78 K) in HTS films and coatings based on RE-Ba-Cu-O (RE = Y, Gd, Nd, Sm) reaches 10^6 – 10^7 A/cm² [2, 3]. An active work is continued to improve the technology for obtaining HTS films and coatings in order to increase the critical current and reduce its dependence on magnetic field and thickness of the superconducting layer [4–6].

The value of j_c in superconductors depends on the ability of material to pin Abrikosov vortices, whose motion leads to the emergence of a resistive state and electrical resistance [7–9]. It is widely believed that the most effective vortex pinning centres are extended linear (columnar) defects with a radius of order the coherence length, ξ , of the superconducting state (for HTS materials, ξ is of about a few nanometers). The efficiency of such linear pinning centres is determined by their ability to pin a vortex along its entire length, when magnetic field is directed along the axis of the linear defect [3, 8, 10]. In HTS materials, effective linear vortex pinning centres are: artificially created columnar defects, which usually arise, when material is irradiated with heavy ions with high energies of 1–10 GeV order (see, for example, review [10]); edge and screw dislocations formed during growth of epitaxial HTS films [11–13], as well as so called ‘nanorods’ formed due

to the self-organization of nanosize dielectric particles implanted inside the HTS material (*e.g.*, BaZrO₃, Y₂O₃, *etc.*) [2–6]. It should be noticed that, for other superconducting materials, *e.g.*, for a new class of superconductors, namely, ferropnictides, which are currently being actively studied, linear extended defects also play a significant role in enhancing the pinning of vortices and increasing the critical current [14–16]. In sufficiently weak magnetic fields, the number of Abrikosov vortices is significantly less than the number of linear defects, and the intervortex distance is significantly larger than the radius of a cylindrical linear defect. In this case, the interaction of vortices with each other can be neglected in comparison with the pinning force created by the linear defect. The type of the pinning potential well $U_p(r)$ for a vortex parallel to the axis of cylindrical linear defect was calculated in a number of theoretical models [17–21]. The state and characteristics of the superconductor are then determined by the interaction of an individual vortex with the pinning centre and effect of the Lorentz force on it: $\mathbf{F}_L = [\mathbf{j}(r) \times \phi_0]$, where $\mathbf{j}(r)$ is the transport current density, $\phi_0 = h/2e \cong 2.07 \cdot 10^{-15} \text{ T} \cdot \text{m}^2$ is the magnetic flux quantum, which is created by an Abrikosov vortex. The condition for applicability of the single-vortex approximation is following: $B \ll B_\phi = n_d \phi_0$, where $B = n_v \phi_0$ is magnetic induction in the superconductor, n_v and n_d are two-dimensional concentration of vortices and linear defects, respectively. Typical values for B_ϕ are in the range of 1–10 T [10–13]. So, the single-vortex approach can be used for a variety of problems, concerning the role of columnar pinning sites. Within the framework of such a ‘single-vortex’ approach, which does not take into account the interaction of vortices with each other, it was shown that the escape of vortices from linear defects and their subsequent dynamics at finite temperatures in the presence of transport current with a density j occurs due to the mechanism of fluctuation formation of partially depinned vortex loops in the bulk of the superconductor [8, 11, 22, 23] or vortex ‘tongues’ near the surface [24] with a size exceeding a certain critical value $l_c(j)$ and, accordingly, an excitation energy greater than the critical value $U_c(j)$: $l_c(j) \propto U_c(j) \propto 1/j$. Such loop excitations are sources for the development of an instability of the pinned state of the vortex filament, since, at $l > l_c(j)$, they start to increase in size under the action of the Lorentz force created by transport current, thereby ensuring the sliding of the vortex from the linear defect and its movement to the adjacent linear defect located in the direction of the Lorentz force. This scenario of disruption (depinning) and subsequent transport of vortices in superconductors with parallel linear defects directed along the c -axis of the crystal is confirmed by a number of experiments performed on HTS single crystals at not too high magnetic fields and temperatures: $T < T_{BG}$ and $B < B_\phi$ (T_{BG} is the critical temperature for the vortex ensemble transition to the ‘Bose glass’ state, $B_\phi = n_d \phi_0$ is the

characteristic ‘matching field’, n_d is the density of columnar defects per unit surface of the sample) [8, 9]. Meanwhile, the problem of disruption of a single vortex and depinning critical current value in the limit of low temperatures at $T \rightarrow 0$ K, when thermal fluctuations and processes of thermal excitation of vortex loops of critical size can be neglected, still needs its further exploration.

2. PROBLEM STATEMENT

In this paper, we study the dynamic behaviour of an elastic Abrikosov vortex filament in a pinning potential well of a linear defect in a 3D anisotropic superconductor under the action of a non-uniformly distributed and non-stationary transport current. We also calculate the corresponding depinning critical current and its dependence on the vortex elasticity and sample thickness. The new feature first considered in the present work is the time of vortex depinning from linear defect. Its dependences on the sample thickness as well as the transport current value are numerically calculated. We consider the case of a plate (film) of a moderately anisotropic layered superconductor (REBCO type) with linear defects passing through the entire thickness of the plate and oriented along the crystal axis c , perpendicular to the two-dimensional superconducting layers and the surface of the sample. It is assumed that the plate thickness $d > 2\lambda$, where $\lambda \equiv \lambda_{ab}$ is the London penetration depth of a weak magnetic field. The instability threshold of the pinned state of the vortex and the corresponding values of the depinning critical current for different values of thickness d in the range of $(2-10)\lambda$ are found. The study of the stability of the vortex pinning state is carried out taking into account the elastic properties of the vortex filament and the non-uniformity of the distribution of the Lorentz force from the transport current, acting on the scale of the London penetration depth λ . The shape of the bended vortex filament in the pinning potential of linear defect $U_p[s(z)]$ and additionally exerted to the action of inhomogeneous Lorentz force $F_L(z) = \phi_0 j(z)$ is described by its displacement $s(z)$ relative to the defect axis (the Oz -axis) directed perpendicularly to the plate. In this case, the equilibrium shape of the vortex filament can be obtained by minimizing its energy functional [23]:

$$W\{s(z)\} = \int_{-d/2}^{d/2} \Phi(z) dz, \quad (1)$$

$$\Phi(z) = \frac{P}{2} \left(\frac{\partial s}{\partial z} \right)^2 - \phi_0 j(z) s(z) + U_p[s(z)]. \quad (2)$$

Here, P is the vortex line tension: $P = E_\phi / I^2$, $E_\phi = \phi_0 H_{c1}$ is the vortex

self-energy per unit length of the vortex line, Γ is anisotropy coefficient for layered material (like REBCO) [9, 23], H_{c1} is the lower critical field; for pinning potential $U_p(s)$, we use the Lorentzian form [20, 26]:

$$U_p(s) = -U_{p0} \frac{r_p^2}{r_p^2 + s^2}, \quad (3)$$

where the pinning well depth, U_{p0} , is of order of the vortex self-energy [23], r_p is a characteristic size of the pinning potential well created by the columnar defect: $r_p > \xi$ (ξ is the coherence length).

Minimization procedure applied to Eq. (1) gives the following equation for the equilibrium form of the vortex line [23, 25]:

$$P \frac{d^2 s}{dz^2} + \phi_0 j(z) - \frac{dU_p}{ds} = 0 \quad (4)$$

with boundary conditions $(ds/dz)_{z=\pm d/2} = 0$, which mean that the bended vortex line on the surface should be oriented normally with respect to the plate surfaces. Based on Eq. (4), the form of the bended vortex line in the presence of transport current with an inhomogeneous density $j(z)$ and stability threshold for equilibrium solutions of Eq. (4) were explored and the depinning critical current was examined in Refs. [25, 27].

In the present work, we investigate dynamics of elastic vortex filament in the pinning potential well of linear defect under the influence of nonstationary transport current. More precisely, we explore dynamics of the vortex escape from linear pinning site under the action of inhomogeneously distributed transport Meissner current $j(z, t)$, which is switched on at some moment t_0 :

$$j(z, t) = \Theta(t - t_0) j_0 \frac{\cosh(z / \lambda)}{\cosh(d / (2\lambda))}. \quad (5)$$

Here, $\Theta(t - t_0)$ is the unit step (Heaviside) function; j_0 is amplitude of the Meissner transport current concentrated mainly near both plates surfaces on the scale of the London penetration depth λ .

To explore the dynamics of a pinned vortex line under the nonstationary Lorentz-force action determined by the current density distribution given by Eq. (5), we use the Lagrange dynamics equation. Based on Eqs. (1), (2), the latter can be written in form:

$$P \frac{\partial^2 s}{\partial z^2} + \phi_0 j(z, t) - \frac{\partial U_p}{\partial s} = \eta \frac{\partial s}{\partial t}. \quad (6)$$

Here, η is viscosity coefficient for the vortex filament (per its unit

length): $\eta = \frac{\phi_0 B_{c2}}{\rho_n}$ (B_{c2} is the upper critical field; ρ_n is the normal state resistivity); the pinning potential $U_p(s)$ is given by Eq. (3). We explore Eq. (6) with boundary conditions similar to that used above for Eq. (4) and time-dependent current given by Eq. (5). To solve Eq. (6), we use dimensionless variables and coefficients, which enter in this equation, as given by the following transformations:

$$s \rightarrow \frac{s}{r_p}; z \rightarrow \frac{z}{\lambda}; d \rightarrow \frac{d}{\lambda}; t \rightarrow \frac{0,01\alpha_L}{\eta} t; j_0 \rightarrow \frac{j_0}{j_{c0}}; P \rightarrow \frac{P}{\alpha_L r_p} \left(\frac{r_p}{\lambda} \right)^2. \quad (7)$$

Here, α_L is the so-called Labusch parameter defined as follows:

$$\alpha_L = \left. \frac{d^2 U_p(s)}{ds^2} \right|_{s=0}. \quad \text{It characterizes the rigidity of the pinning potential}$$

well near its bottom. For the pinning potential given by Eq. (3), one

gets: $\alpha_L = \frac{2U_{p0}}{r_p^2}$. The Labusch parameter together with the radius of

the pinning potential well, r_p , define a characteristic scale of the depinning critical current density, $j_{c0}: j_{c0} = \alpha_L r_p / \phi_0$ [25, 27].

Now, using these dimensionless variables (7) and renormalized coefficients, we can rewrite the dynamic Eq. (6) in the following form:

$$P \frac{\partial^2 s}{\partial z^2} + j_0 \Theta(t - t_0) \frac{\cosh(z)}{\cosh(d/2)} - \frac{s}{(1+s^2)^2} = 0.01 \frac{\partial s}{\partial t}. \quad (8)$$

Here, s, z, t are dimensionless variables, and P, j_0, d are proper dimensionless parameters defined by the transformation (7); $\Theta(t - t_0)$ is the step-like (Heaviside) function.

The obtained results for vortex dynamics based on the numerical solution of Eq. (8) with a proper choice of parameters, which rule the vortex dynamics, are presented in the next Section.

3. RESULTS OF NUMERICAL MODELLING

In this section, we investigate an occurrence of current-driven instability of the pinned vortex state, and employing the numerical solution of the dynamic Eq. (6), calculate the corresponding depinning critical current density, at which the vortex filament starts its escape from the columnar defect under the action of switched on (at some moment t_0) an inhomogeneous Lorentz force, concentrated near the surface within the screening layer of width λ .

In Figure 1, the evolution of the vortex filament shape is shown af-

ter the driving current switching on at the moment t_0 . This figure illustrates the case of low current densities (5): $j_0 < j_c$ (j_c is the depinning critical current density on the plate's surface). One can see that, after the transition process, the vortex line shape acquires the stationary form and its time evolution ceases. At high current densities, $j_0 > j_c$, the depinning process for a flexible vortex filament starts at the plate surface and then the vortex detachment wave propagates from the surface to the specimen centre. During this transition process, the vortex line distortion at the plate centre ($z=0$) increases and then begins to exceed the pinning potential well radius, *i.e.*, when $s(0, t) > r_p$, the vortex filament elaborates from the pinning potential well and moves freely driven by the inhomogeneous Lorentz force counteracted by the viscous friction. This type of vortex filament dynamics caused by the step-like switching on the driving Lorentz force, caused by the inhomogeneous transport current density $j(z, t)$ given by Eq. (5) is shown in Fig. 2.

It worth noticing that the vortex depinning process in this case has a definite duration, which can noticeably effect on electromagnetic response of a superconductor. For instance, it can cause a delay in a switching-on of the resistive state at the pulse current loading of this type superconductor.

The time evolution of the vortex ends position on the plate surface, $s(\pm d/2, t)$ together with the position of the vortex filament centre, $s(0, t)$, obtained by numerical solution of Eq. (8) for current value larger than the critical, $j_0 > j_c$, and proper values of parameters entering this equation, is demonstrated in Fig. 3. This figure distinctly demon-



Fig. 1. 3D graph for vortex filament deviation $s(z, t)$ from the columnar defect caused by switched on at the moment t_0 an inhomogeneous Lorentz force corresponding to the current density $j(z, t)$ given by Eq. (5). This graph is obtained by numerical solution of Eq. (8) for proper parameters values.

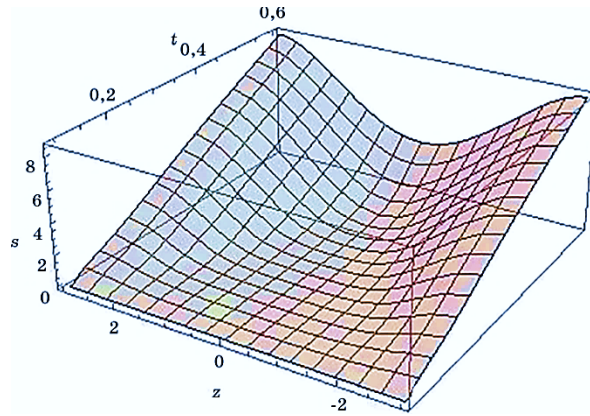


Fig. 2. 3D graph of the pinned vortex filament escape from the columnar defect at current higher than the depinning critical value.

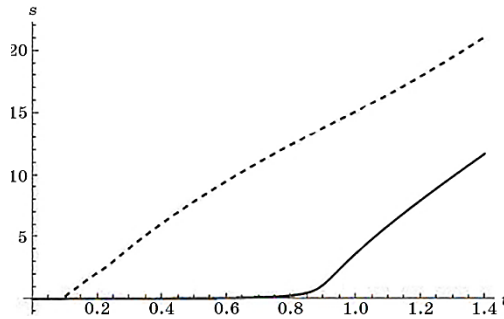


Fig. 3. The time evolution of the vortex ends position on the plate surface, $s(\pm d/2, t)$, together with the position of the vortex filament centre, $s(0, t)$, obtained by numerical solution of Eq. (8) for the current amplitude larger than the critical one, $j_0 > j_c$, and proper values of parameters.

strates that the depinning process caused by the step-like switching on the driving current described by Eq. (5) starts on the surface and propagates inside the plate leading to the some delay of the vortex central part escape from the columnar defect.

In what follows, we consider the delay time for the vortex depinning process, τ , as that, which is necessary for the achievement of the deviation of the vortex central part, $s(0, t - t_0)$, the value of the pinning potential well radius, r_p : $s(0, \tau) = r_p$. Corresponding dependences of the delay time on the sample thickness, $\tau(d)$, and transport current amplitude value, $\tau(j_0)$, are shown in Fig. 4 and Fig. 5, respectively, for proper values of parameters in Eq. (8).

In addition, we have calculated numerically the average critical current density and its dependence on the plate thickness d . This plot is

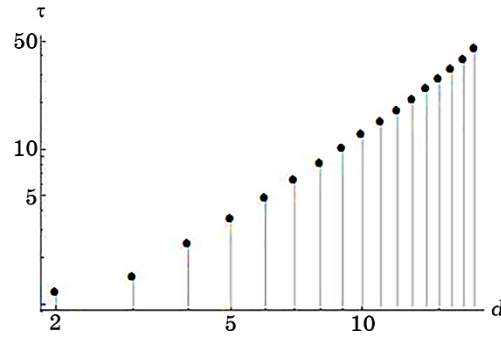


Fig. 4. Thickness dependence of the delay time for vortex escape from columnar defect, $\tau(d)$, at $j_0 > j_c$ calculated from Eq. (8) for the following values of parameters in Eq. (8): $P = 0.1$, $j_0 = 0.7$, $j_c = 0.5$.

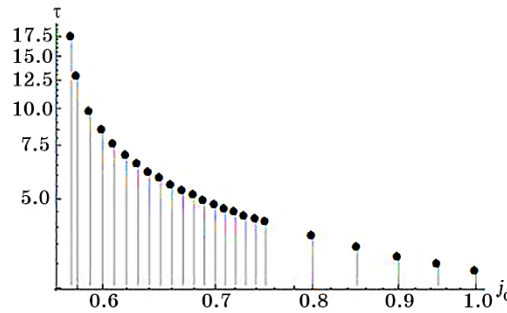


Fig. 5. Dependences of the delay time on the transport current amplitude value, $\tau(j_0)$, for proper values of parameters in Eq. (8): $P = 0.12$, $d = 8$, $j_c = 0.5$.

demonstrated in Fig. 6.

The descending dependence $\bar{j}_c(d)$ means that the total critical current through the whole cross-section of the plate will increase slower than simple linear proportional dependence on d . So, for achievement of high current-carrying abilities of superconducting tapes and coatings produced from HTS materials, the presence of additional pinning sites of other type (point-like strong pinning sites or columns splitting) is necessary [3–5].

4. CONCLUSION

In this work, the mechanics of vortex escape from a columnar defect and necessary conditions for vortex depinning were investigated theoretically by use of a nonlinear dynamic equation, which describes the behaviour of an elastic vortex filament settled in the pinning potential

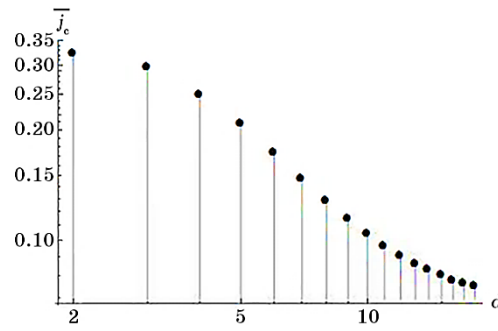


Fig. 6. Depinning critical current density averaged of the plate thickness.

well and exerted to the action of inhomogeneously distributed Meissner current, which is switched on at some moment t_0 in a step-like way. The time evolution of the vortex filament form during the depinning process was modelled by the solution of the nonlinear dynamic equation, and the results of this modelling are presented in the form of 3D graphs. The most significant physical result obtained in the present work is a demonstration of the finite duration of the depinning process. We have numerically calculated the depinning time of a single vortex filament from a columnar defect, which characterizes the delay in the response of vortices on alternating current and the dependences of this delay time on the specimen thickness and the driving current value. The corresponding effect of this type of delay of the vortex response on the electrodynamics of a superconductor with columnar defects and its manifestation on electrodynamic characteristics will be studied elsewhere.

AUTHORS' CONTRIBUTIONS

O. S. Hrechykha solved numerically nonlinear differential equations, plotted corresponding graphs for curved vortex lines and calculated physical characteristics. A. L. Kasatkin supervised the project, formulated the main conceptual ideas, elaborated analytical approaches, and wrote the manuscript with input from all authors. V. P. Tsvitkovskiy performed numerical calculations of physical characteristics, verified analytical approaches, and reviewed the literature. All authors approved the final version of the manuscript.

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