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Review of Modern Practices for Ensuring Strength and Durability Under Multiaxial Loading

M. V. Karuskevych[*], T. P. Maslak[*], T. V. Turchak[**], and O. M. Karuskevych[*]

[*]*National University 'Kyiv Aviation Institute',*
1 Lyubomyr Huzar Ave.,
UA-03058 Kyiv, Ukraine
[**]*G. V. Kurdyumov Institute for Metal Physics, N.A.S. of Ukraine,*
36 Academician Vernadsky Blvd.,
UA-03142 Kyiv, Ukraine

The article combines theoretical and applied aspects of the multiaxial fatigue problem. It presents a brief overview of modern theories and hypotheses, as well as examples of methods, for quantitatively assessing fatigue damage in metallic structures under multiaxial loading. The necessity of conducting this analysis arises, firstly, from the importance and prevalence of the issue of multiaxial cyclic loading, and secondly, from the extensive number of attempts to address it. The analysis includes characteristics of classical approaches, contemporary models of multiaxial-fatigue damage, and examples of normative documents and methodologies, which have found broad practical application. The aim is to synthesize information on methods for determining damage accumulation under multiaxial loading to identify their potential for further development and application in ensuring the strength and durability of aerospace structural elements. As shown, existing methods for assessing accumulated fatigue damage and predicting durability under multiaxial cyclic loading can be divided into two groups: those ones effective for proportional loading and those ones effective for more complex, non-proportional loading, including in-phase and out-of-phase conditions. Although methods developed for non-proportional loading, particularly based on the critical-plane concept, take into account the geometry of the localized

Corresponding author: Mykhailo Karuskevych
E-mail: mykhailo.karuskevych@npp.nau.edu.ua

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damage plane, they do not consider the crystallographic slip direction on the critical planes. Accounting for rolling texture, as shown while considering aviation aluminium alloy 2024 T3, can bring the Huber–Mises approach for multiaxial loading closer to the real processes occurring in structural materials at the microscale level.

Key words: aerospace structures, multiaxial fatigue, strength theories, proportional loading, non-proportional loading, equivalent stresses, critical plane, irregular loading, crystallographic factor.

У статті поєднано теоретичні та прикладні аспекти проблеми багатовісної втоми. Представлено короткий огляд сучасних теорій і гіпотез, а також приклади методів кількісної оцінки пошкоджень через втому у металевих конструкціях за багатовісного навантаження. Необхідність проведення цієї аналізи виникає, по-перше, через важливість і поширеність питання багатовісного циклічного навантаження, а по-друге, через велику кількість спроб його вирішення. Аналіза включає характеристики класичних підходів, сучасні моделі пошкоджень від багатовісної втоми, а також приклади нормативних документів і методологій, які знайшли широке практичне застосування. Метою є синтеза інформації про методи визначення накопичення пошкоджень за багатовісного навантаження, щоб визначити їхній потенціял для подальшого розвитку та застосування у забезпеченні міцности та довговічности елементів аерокосмічних конструкцій. Показано, що наявні методи оцінки накопичених пошкоджень від втоми та прогнозування довговічности за багатовісного циклічного навантаження можна розділити на дві групи: ефективні для пропорційного навантаження й ефективні для більш складного, непропорційного навантаження, включаючи синфазні та нефазні умови. Хоча методи, розроблені для непропорційного навантаження, зокрема засновані на концепції критичної площини, враховують геометрію площини локалізованого пошкодження, вони не враховують напрямок кристалографічного ковзання на критичних площинах. Врахування текстури кочення, як це показано за розгляду авіяційного алюмінійового стопу 2024 Т3, може наблизити підхід Губера–Мізеса щодо багатовісного навантаження до реальних процесів, що відбуваються у конструкційних матеріялах на мікрорівні.

Ключові слова: аерокосмічні конструкції, багатовісна втома, теорії міцности, пропорційне навантаження, непропорційне навантаження, еквівалентні напруження, критична площина, нерівномірне навантаження, кристалографічний фактор.

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1. INTRODUCTION

The necessity of analysing and systematizing information presented in published articles on methods for predicting the durability of structural elements operating under multiaxial loading arises from the complexity and persistent relevance of the multiaxial fatigue problem.

The complexity of multiaxial loading, damage, and failure has led to the emergence and development of numerous theories, calculation methods, and experimental studies. Existing analytical methods differ in their practical applicability, which is largely determined by their key parameters and the limitations imposed by the physical and mechanical properties of materials and the characteristics and conditions of their loading.

Multiaxial cyclic loading and the corresponding fatigue damage are classified as proportional and non-proportional, in-phase and out-of-phase. An example of a structure subjected to multiaxial cyclic loading is a modern aircraft.

Loading in aircraft components, particularly the fuselage arises from: excessive pressure in the cabin, causing axial and radial stresses; loads transferred from the wings and empennage during flight; landing gear loads not only during landing but also during ground movements.

It is noteworthy that the proportional loading caused by cabin pressurization combines with aerodynamic forces leading to bending and torsion of the aircraft structure. These loads are irregular and non-proportional.

Simultaneously, the stringent airworthiness requirements for aircraft, as reflected in relevant international airworthiness standards, must be emphasized. These standards demand detailed analysis and robust methodologies for fatigue life prediction [1].

Not all modern developments of failure and damage models under multiaxial loading have found practical application or representation in normative documents. However, their contribution to the advancement of the science of multiaxial loading undoubtedly has a cumulative effect.

2. STANDARDS AIMED AT SOLVING PRACTICAL PROBLEMS OF ENSURING STRENGTH AND DURABILITY UNDER BIAXIAL LOADING

Ensuring strength and durability under biaxial loading is a common task for strength specialists in many fields of engineering. This has led to the development and application of numerous standards for performing calculations and making decisions. Typically, these standards are based on practices that have proven their effectiveness. Below are some examples that provide an understanding of the widespread use of standardized, verified approaches to addressing multiaxial loading issues. Methods for determining equivalent and permissible stresses under static loading are covered in the Ukrainian State Standard DSTU 2464-94 [2].

The technical report of the International Organization for Standardization (ISO/TR) [3] addresses the fundamental existing principles of testing and analysing fatigue damage under multiaxial loading. It presents experimental data accumulated over 80 years, with the initial studies conducted in the 1930s. The evolution of methodologies has progressed from investigating specific conditions of biaxial loading, such as torsion, bending with torsion, cantilever bending, and loading with internal pressure, to universal methods employing two standard types of specimens: tubular and cruciform, along with the corresponding designs of testing machines.

The main methods of fatigue testing under multiaxial loading are as follow.

1. Bending with torsion. This type of testing was the first methodology for combined loading in the field of high-cycle fatigue at room temperature. Between 1930 and 1950, numerous machines were developed to study the fatigue of steels used in aviation engines, particularly for crankshafts.
2. Axial loading with torsion.
3. Axial loading with internal and external pressure.
4. Axial loading combined with internal pressure, external pressure, and torsion.
5. Low-cycle loading of cruciform specimens.
6. Loading of cruciform specimens with a stress concentrator in the central zone of the specimen.

The same document discusses modern methods for analysing fatigue damage under multiaxial loading. When designing structural elements operating under multiaxial cyclic loading, the finite element method holds a well-justified and established position for determining stresses and strains.

Tresca's yield criteria (maximum shear stress criterion) and Huber–Mises criteria (octahedral shear strain criterion), combined with the linear Palmgren–Miner rule, are used to estimate fatigue life. However, these approaches are not always supported by experimental evidence.

The ASTM E2207-15 standard [4] specifies the methodology for testing materials under combined axial and torsional loading. The standard addresses strain-controlled testing for axial, torsional, combined proportional, and non-proportional combined loading using thin-walled tubular specimens under isothermal conditions, as well as at room and elevated temperatures. Figure 1 explains the terms 'in-phase' and 'out-of-phase' loading.

The recommended dimensions of the thin-walled tubular specimen are shown in Fig. 2.

It is worth noting that the standard emphasizes the geometric features in connection with the material microstructure. The wall thickness of the specimen must be sufficient to prevent stability loss under

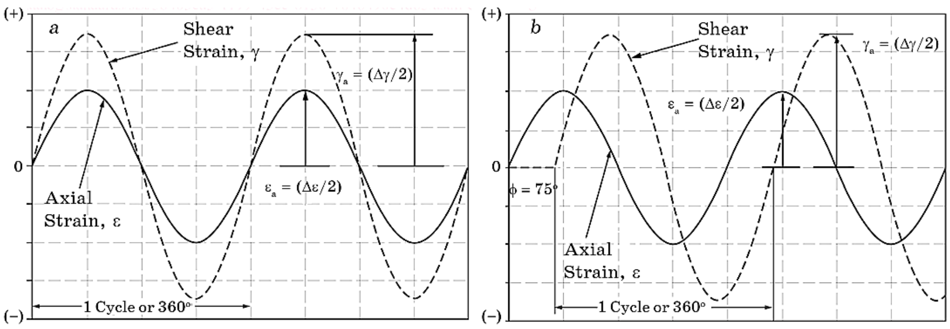

Fig. 1. The schematic of axial and shear strain for in-phase and out-of-phase loading under tensile–torsion testing conditions.

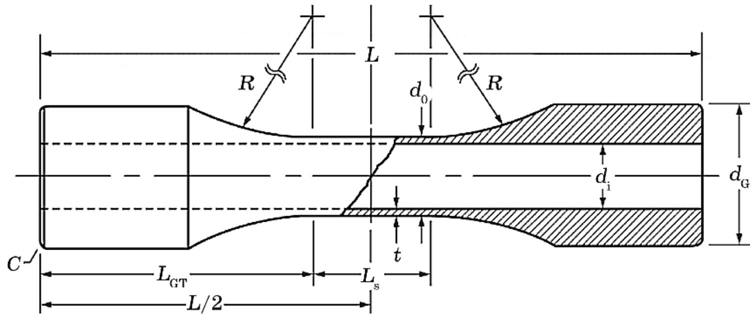

Fig. 2. Thin-walled tubular specimen for combined testing under axial force and torsion.

cyclic loading. At the same time, to meet the definition of a 'thin-walled tube', the diameter-to-wall-thickness ratio must be at least 10:1. Besides, there are microstructural requirements: the wall thickness of the tube must contain at least 10 grains, ensuring isotropy.

The standard, jointly developed by the API (American Petroleum Institute) and ASME, addresses multiaxial stresses in pipelines and pressure equipment. The standard incorporates the critical plane method proposed by Brown and Miller, the 'strain–life' equation, and the method of incremental correction of the material's stress–strain state proposed by Neuber, utilizing the nonlinear kinematic hardening model by Chaboche.

In many cases, existing standards include finite element analysis to model complex stress states, as exemplified in Refs. [10–13].

Regulatory documentation for conducting experimental studies does not provide for the implementation of all possible combinations of principal stresses, which are infinite. As a result, the task of predicting durability under multiaxial loading is addressed using various analytical methods that account for the specific characteristics of the loading and materials.

The foundation of many analytical methods lies in classical theories formulated for predicting static strength.

3. CLASSICAL STRENGTH THEORIES

The first studies and hypotheses date back to the works of outstanding scientists of the past, Leonardo da Vinci (1452–1519) and Galileo Galilei (1564–1642), who initiated research on the strength of materials and structures. With the development of materials science, strength theories and the corresponding criteria have been further developed. Today, the primary, *i.e.*, fundamental strength theories are the first, second, third, fourth, and fifth (Mohr's theory).

According to the first strength theory, strength under complex stress states is determined by the maximum normal stresses. The authors of the first strength theory are G. Galilei and W. Rankine (1820–1872). This theory does not take into account the effect of the other two principal stresses and is proved effective only for brittle materials.

The second strength theory was proposed by E. Mariotte (1620–1684) and later developed by B. Saint-Venant (1797–1886). According to the second strength theory, if the first principal strain reaches the limit value of uniaxial tension, this leads to failure. The theory was used for some time in the XIXth century, but insufficient justification and inconsistency of application results led to the search for more effective approaches to predicting the limit state.

According to the third strength theory, the limit state is caused by reaching a critical value of the maximum shear stress. The theory was proposed by Henri Tresca (1814–1885). According to the proposed maximum shear stress criterion, the condition for reaching the limit state is a critical value of the equivalent stress, which is defined as the difference between the algebraic maximum and minimum principal stresses. For plastic materials, this has been experimentally confirmed, but an obvious drawback is the neglect of the second principal stress.

The fourth strength theory has found the widest application in strength calculations under complex stress states, especially under static loading. The strength criterion is the specific potential energy of distortion accumulated during deformation. The theory was developed by Huber (1872–1950) and Mises (1883–1953). Its theoretical justification was found in the works of G. Hencky (1885–1951); so in scientific literature, this criterion is often referred logically to as the 'Huber–Mises–Hencky criterion'.

In modern computer-aided design and analysis systems, which are based primarily on the finite element method and determine the parameters of the stress–strain state of structures, the corresponding equivalent stresses are referred to as Mises's stresses.

According to the Huber–Mises theory, the critical state occurs when the potential energy of distortion reaches a critical value.

It should be noted that the Huber–Mises criterion assumes identical material properties in all directions; however, a significant number of metals are anisotropic. For example, rolled sheets of aluminium alloys exhibit anisotropy as a result of deformation during the manufacturing process of such sheets. In this case, anisotropy may be related not only to changes in the shape of metal grains but also to the formation of rolling texture, which is determined by preferred crystallographic slip directions and preferred crystallographic slip planes.

The fifth strength theory was proposed by Mohr (1835–1918). Mohr's theory is most effective for materials that exhibit different resistance to tension and compression. According to Mohr's theory, allowable stresses depend on the relationship between the components of the stress state, which is determined graphically using Mohr's circles. Mohr's theory is most effective in determining the limit state of brittle materials and requires a significant experimental database. If the allowable stresses in tension and compression are the same, the calculation results according to Mohr's theory coincide with the third strength theory.

The first studies of multiaxial stress states under monotonic loading considered the classical strength theories of Lamé and Tresca from the late XIXth century and the Huber–Mises model from the early XXth century. According to the Huber–Mises formula, the equivalent stress can be found as

$$\sigma = \sqrt{0.5[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2]} + \sqrt{3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}\,,$$

where σ_{eq} is equivalent Huber–Mises stress; σ_x is longitudinal (axial) stress; σ_y is hoop stress; σ_z is transversal stress; τ_{xy}, τ_{yz}, τ_{zx} are shear stress components.

Unfortunately, the direct application of the aforementioned strength theories for fatigue analysis is not always sufficiently effective.

While the first combined tests date back to the work of Lanza in 1886 [14], systematic studies of multiaxial fatigue were conducted by Gough and Pollard in the 1930s [15]. Their research on fatigue under combined bending–torsion loading created the foundation for models proposed by Gough [16], Sines [17], and Findley [18] in the 1950s. In most early studies of multiaxial fatigue, plastic deformation was generally insignificant.

The study [19] demonstrated that classical yield criteria, such as the Huber–Mises distortion energy criterion, are often applied for predict-

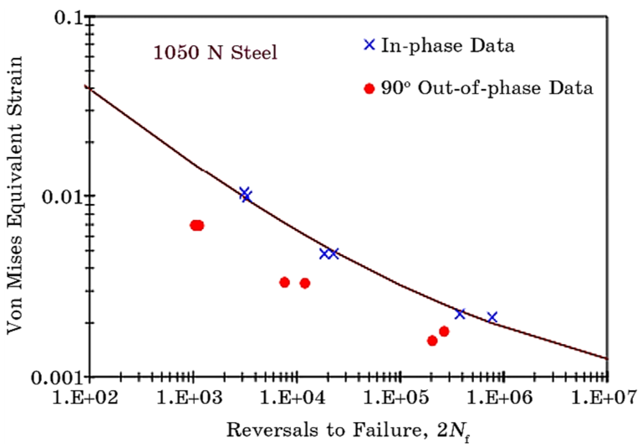

Fig. 3. Fatigue curves for 1050N steel under proportional and non-proportional cyclic loading.

ing multiaxial fatigue.

Such criteria may be effective for in-phase or proportional loading, but they do not account for the reduction in durability observed under out-of-phase or non-proportional loading. For example, in the case of out-of-phase sinusoidal axial–torsional loading with a phase shift of 90°, where axial stress is related to torsional stress through $\sqrt{3}$, the Huber–Mises equivalent stress does not change during the loading process. This should theoretically lead to unlimited durability, which highlights the limitations of this criterion and its unsuitability for out-of-phase or non-proportional loading.

The reduction in durability under non-proportional loading is explained by additional deformation-induced hardening. Under non-proportional loading, the loading components change independently. In this case, not only do the magnitudes of the principal stresses change during loading, but their directions also vary.

The comparison of fatigue durability for in-phase and out-of-phase (90°) loading of T1050 steel is shown in Fig. 3 [20]. Titanium alloys do not exhibit strengthening under non-proportional loading, whereas for steel, the strengthening is in the range of 10–15%. The dependence of the number of load cycles on the Huber–Mises equivalent stress for 1050N steel is shown in Fig. 3.

4. MODERN FATIGUE DAMAGE MODELS FOR MULTIAXIAL CYCLIC LOADING OF METALLIC STRUCTURES

For high-cycle in-phase fatigue of ductile materials, the equivalent stress criteria of Tresca and Huber–Mises, initially proposed for static loading, have found some application, while the Rankine's criterion is used for brittle materials. However, these criteria are not effective for low-cycle fatigue analysis.

A range of equivalent stress criteria is reviewed in Ref. [21], including the Manson–McKnight method and its modifications. This method involves using equivalent strain energy to assess fatigue levels. The total energy of plastic and elastic deformations is considered decisive for crack initiation and growth processes. The method employs a criterion that accounts for both normal and shear stresses, enabling more accurate evaluation of crack initiation conditions under multiaxial loading. The Manson–McKnight method has been applied in aviation, particularly for predicting the durability of aviation engine components [22].

Methods based on the concept of a critical plane, initiated by Findley [18], are increasingly being applied and incorporated into regulatory documents for fatigue calculations under multiaxial loading. Findley's fatigue criterion focuses on stresses acting on specific planes, referred to as critical planes; these are the planes where the maximum damage criterion is reached. A critical plane is defined as the plane where an extreme combination of shear stresses/strains and normal stresses/strains occurs. The fatigue process is governed by the combination of stresses and strains acting on the critical plane. Findley's model is based on the observation that cyclic shear stresses lead to crack initiation in ductile materials, while normal stresses influence their propagation.

In general form, this model has a view $\max(\tau_{a,n} + k\sigma_n) = f$, where $\tau_{a,n}$ is the shear amplitude on a plane with a unit normal **n**, *i.e.*, $\tau_{a,n} = \Delta\tau_n / 2$, and σ_n is the normal stress on the same plane. The two material parameters k and f can be determined from two fatigue-loading tests.

Brown and Miller [6] expanded this theory by demonstrating the role of both shear strain and normal one on the maximum-shear plane. They introduced the concept of constant-life contours defined by principal strains (ε_1, ε_2, ε_3) and graphically represented by the equation

$$\frac{\varepsilon_1 - \varepsilon_3}{2} = f\left[\frac{\varepsilon_1 + \varepsilon_3}{2}\right].$$

Their work initiated widespread adoption of a new approach, namely, the critical plane concept, which has been further developed in academic research and practical applications for evaluating durability under multiaxial cyclic loading. The study demonstrated how the orientation of stress components affects fatigue damage and failure.

The critical plane concept is used for both proportional and non-proportional loading, but it is particularly valuable for analysing non-proportional cyclic loading, where equivalent stress models are not always effective.

Fatigue damage models based on the critical plane concept are currently considered the most effective for analysing multiaxial cyclic loading. In Ref. [23], the effectiveness of several critical plane models was examined, including the Wang–Brown model, the Fatemi–Socie model, the Smith–Watson–Topper model, and others. These models were evaluated under both proportional and non-proportional multiaxial loading conditions for a wide range of durability, covering both the low-cycle fatigue zone and the high-cycle fatigue zone, and for various metals. The Wang–Brown and Fatemi–Socie approaches were found to be the most accurate.

The Fatemi–Socie model [24] extends the Brown's and Miller's model to predict multiaxial fatigue under in-phase and out-of-phase loading. It is based on the premise that maximum shear strain governs crack initiation, while normal stress must be considered in the damage parameters to account for the additional strengthening effect under non-proportional cyclic loading. The Fatemi–Socie model uses the amplitude of shear strain and the maximum normal stress on the critical plane as fatigue damage parameters:

$$\frac{\Delta\gamma_{\max}}{2}\left(1+k\frac{\sigma_{n,\max}}{\sigma_y}\right)=\frac{\tau_f'}{G}\left(2N_f\right)^{b_\gamma}+\gamma_f'\left(2N_f\right)^{c_\gamma},$$

where $\Delta\gamma_{\max}/2$ is maximum amplitude of shear; $\sigma_{n,\max}$ is maximum normal stress in the critical plane; σ_y is yield limit; G is shear modulus; b_γ, c_γ, τ_f', γ_f' are coefficients of the Coffin–Manson equation for pure torsion at symmetrical cycle; k is correction coefficient.

In work [25], a comparison of several equivalent stress methods and critical plane approaches was conducted. The criterion of absolute maximum principal stress and the signed Mises's criterion were chosen as equivalent stress methods due to their widespread use in industry, attributed to their simplicity and computational speed.

A signing is used to reflect the real load spectrum:

$$\sigma_{eq}=(\mathrm{sign})\frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_1-\sigma_3\right)^2}\,.$$

It is mentioned in the work [25] that there exist different approaches for signing procedure: signing should be applied according to principal stresses or by the sign of the first stress invariant.

The Findley's and Matake's criteria were selected for comparison because they have similar formulations but differ in the definition of the critical plane. Both methods yield excessively non-conservative results. It was concluded [25] that the results strongly support the critical-plane concept.

Reviews of multiaxial fatigue calculation methods for structural materials are also provided in works [26–28].

An example of a model aimed at addressing the issue of ensuring the durability of aviation hydraulic pipelines is presented in work [29]. This study proposes an enhancement to the WYT (Weighted Yielding Theory) model, which assumes that different components of the stress state have varying 'weights' (significance) in their contribution to the overall effect, such as yielding or failure. In the modified model, the maximum shear stress τ_{max} is replaced with the maximum absolute shear stress $|\tau|_{max}$:

$$\frac{\Delta\gamma_{max}}{2}\left(1+\frac{|\tau|_{max}}{\tau'_f}\right)+\Delta\varepsilon_n\left(1+\frac{\sigma_{n,max}}{\sigma'_f}\right)=f\left(N_f\right).$$

The service-life prediction results for four material samples demonstrate that the modified model yields better results compared to the original WYT model. The prediction error using the modified model falls within the range of 2σ to 3σ.

The influence of anisotropy in multiaxial loading models based on equivalent stress approaches is most prominently represented in the Hill's and Barlat's models [30, 31].

Regarding classical critical plane models, such as by Findley's and Matake's ones, they do not account for anisotropy, which can lead to errors in predicting the fatigue behaviour of materials with pronounced texture, such as rolled aluminium alloys. This necessitates the development of modified models and their experimental verification.

V. T. Troshchenko has made a significant contribution to the study of metal fatigue, including the development of fatigue failure criteria and methods for assessing strength under multiaxial loading. For instance, in work [32], deformation and energy criteria for metal fatigue failure were formulated and experimentally validated. Methods for accelerated determination of fatigue limits based on deformation and energy criteria were analysed, and approaches for predicting fatigue resistance characteristics of metals were presented. These approaches take into account the effects of stress concentration, complex stress states, loading modes, and the presence of fatigue cracks.

5. ASSESSMENT OF ACCUMULATED FATIGUE DAMAGE UNDER IRREGULAR MULTIAXIAL LOADING

To evaluate accumulated fatigue damage under irregular multiaxial cyclic loading, it is necessary to have load history data, appropriate damage criteria, and a summation rule for accumulated damage.

The calculation of accumulated fatigue damage remains an unresolved challenge, even for uniaxial loading. Regarding multiaxial proportional and non-proportional loading, it can be confidently stated that this problem is far from being solved.

Work [33] provides a detailed review of fatigue damage accumulation theories. Despite the well-known shortcomings of the Palmgren–Miner rule [34], it remains the fundamental model for assessing accumulated fatigue damage under irregular cyclic loading, including multiaxial scenarios. According to it, the failure happens when the sum of relative damage riches 1: $\sum(n_i / N_i) = 1$, where n_i is the number of loading cycles at stress level i; N_i is the number of cycles to failure at stress level i, determined from the Wöhler's curve.

At the same time, non-linear rules for fatigue damage accumulation have been proposed. These aim to correct non-linearity by accounting for the actual sequence of stress cycles and the effect of stresses below the fatigue limit.

In work [35], it is noted that the shortcomings of the Palmgren–Miner rule may be due to the use of an incorrect damage parameter. It was shown that, in the calculation of accumulated fatigue damage under uniaxial loading of stainless steel, the linear summation rule provides sufficiently accurate results if a parameter is used that accounts for both stress and strain, rather than the conventional stress–cycles (S–N) or strain–cycles (ε–N) curves.

In the case of multiaxial fatigue, changes in loading modes can also influence durability. Studies [36, 37] have demonstrated that torsion preceding tension is more damaging than tension preceding torsion. In work [38], this phenomenon was explained by the fact that torsion cycles lead to the initiation of microcracks on planes where normal stresses promote their growth, whereas tensile stresses do not facilitate crack initiation.

At the same time, as demonstrated in work [39], in the case of low-cycle fatigue of titanium, the 'tension–torsion' sequence proved to be slightly more damaging than the 'torsion–tension' sequence.

Significant contributions to addressing this issue are the reviews presented in works [40, 41].

6. APPLICATION OF SOFTWARE PACKAGES FOR ANALYZING MULTIAXIAL CYCLIC LOADING

The capabilities of various software packages can be demonstrated through practical applications.

In work [42], the study focused on turbine blades of aircraft engines operating under high temperatures and pressures. The finite element method (FEM) implemented in ANSYS was used to model operating conditions and identify critical zones. The Chaboche's model [43], which describes material behaviour under cyclic loading, served as the baseline. For evaluating the fatigue life of turbine blades, models such as Fatemi–Socie [24], Wang–Brown [44], and modified Smith–Watson–Topper [45] were employed. These models assess the strengthening effect under non-proportional cyclic loading and the influence of mean stress in multiaxial fatigue. They are applicable to both proportional and non-proportional loading scenarios.

Work [46] focused on practical cases of fatigue damage analysis using ABAQUS and Fe-Safe. As an example, the study analysed the fatigue damage of a nose landing gear support made of SAE4130 steel. The loading scenario included vertical load, lateral force, and internal pressure in a fluid-gas shock absorber, resulting in multiaxial repeated loading. The fatigue life of the nose landing gear support under multiaxial loading was assessed using the critical plane criterion. The orientation of the critical plane was determined by the Fe-Safe algorithm based on the Brown–Miller model [6], which is widely accepted for SAE4130 steel.

Among software packages for analysing multiaxial fatigue, notable examples also include Nastran [47], LS-DYNA [48], and others.

7. CONSIDERATION OF CRYSTALLOGRAPHIC ORIENTATION IN MULTIAXIAL FATIGUE CALCULATIONS

One of the factors driving the development of the critical plane concept was Findley's observation [18], which noted the crystallographic nature of damage localization. However, the process of microplastic deformation is determined not only by the orientation of the slip plane but also by the slip direction. For face-centred cubic crystals, there are 12 equivalent slip systems, comprising 12 combinations of slip planes in the {111} family and slip directions in the [110] family [49].

It is widely accepted that slip under loading begins in the system with the maximum value of the Schmid's factor, which is determined as $m = \cos(\varphi)\cos(\lambda)$, where ϕ is angle between the axis of loading and perpendicular to the slip plane; λ is angle between the axis of loading and slip direction.

Work [50] demonstrated that accounting for the actual stress components in the slip systems of crystallites when calculating the Mises's equivalent stress under proportional cyclic tension–torsion loading of aluminium alloy 2024 T3 samples improves the accuracy of accumulated fatigue damage assessment and failure prediction. The crystallographic orientation of the crystallites was assumed to follow the texture formed during rolling. This approach was further developed in the work [52] by application for the solving the practical issue: repair of the damaged aircraft structure by the patches oriented for minimizing equivalent stresses at the multiaxially loaded spot of the structure.

However, it should be noted that the maximum Schmid's factor is not the sole determinant in analysing the slip process. For instance, in work [53], it was shown that slip activation can occur in slip systems with Schmid's factors lower than the maximum among the 12 slip systems. Experiments involving tension of zone-refined, ribbon-shaped aluminium single crystals, using the Lang's x-ray topographic method, revealed multiple activated slip systems. At early stages of plastic deformation, slip systems with lower Schmid's factors may activate earlier than those with higher factors, depending on the crystal orientation relative to external surfaces.

8. CONCLUSION

The multitude of theories and models for multiaxial cyclic fatigue damage indicates the absence of a universal approach. The most commonly used methods are equivalent-stress models and critical-plane criteria, each with its own optimal application domain.

Despite well-known criticisms of the Huber–Mises theory, particularly regarding its neglect of anisotropy in metallic structural elements, the equivalent stress formula remains widely used in many methodologies for analysing stress–strain states.

The critical-plane criterion has proven its effectiveness and is widely applied, especially, in automated computations. It has been integrated into software packages designed to address the challenge of damage assessment under multiaxial cyclic loading.

Incorporating anisotropy in metallic materials is advisable at the microstructural analysis level. This approach views the damage process as a result of microplastic slips along slip systems.

AUTHORS' CONTRIBUTIONS

M. V. Karuskevych, T. P. Maslak contributed to the formulation of the problem and methods for its solution. O. M. Karuskevych, T. P. Maslak verified the analytical approaches, conducted a literature review that combines theoretical and applied aspects of the problem of multiaxial fatigue. T. V. Turchak, M. V. Karuskevych verified the analytical approaches, synthesized information on methods for determining damage accumulation under repeated loading, and prepared the manuscript of the article. All authors reviewed and approved the final version of the manuscript.

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